ФЕДЕРАЛЬНОЕ АГЕНТСТВО МОРСКОГО И РЕЧНОГО ТРАНСПОРТА
ФЕДЕРАЛЬНОЕ ГОСУДАРСТВЕННОЕ БЮДЖЕТНОЕ ОБРАЗОВАТЕЛЬНОЕ УЧРЕЖДЕНИЕ ВЫСШЕГО ОБРАЗОВАНИЯ

МОРСКОЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ
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ВЫПУСКНАЯ КВАЛИФИКАЦИОННАЯ РАБОТА

«ИСПОЛЬЗОВАНИЕ АНАЛИТИЧЕСКОЙ ГЕОМЕТРИИ ДЛЯ РЕШЕНИЯ ЗАДАЧ НА МАНЕВРЕННОМ ПЛАНШЕТЕ»

(НА АНГЛИЙСКОМ ЯЗЫКЕ)

USE OF ANALYTIC GEOMETRY FOR TASK SOLUTION ON MANEUVERING BOARD

ВКР. 2028-к.12.02/1-2.26.05.05 ПЗ

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2019

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ЗАДАНИЕ

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ЗАДАНИЕ

НА ВЫПУСКНУЮ КВАЛИФИКАЦИОННУЮ РАБОТУ

Курсанту-выпускнику СВФ

1. **Наименование темы:**

   «Использование аналитической геометрии для решения задач на маневренном планшете» *(на английском языке)*

   Use of analytic geometry for task solution on maneuvering board

2. **Основание для разработки:** Приказ от «01» августа 2018 г. № 2028-к

3. **Технические требования:**

   Особенности РЛС/ САРП

   Применение вычислительной программы

4. **Перечень разрабатываемых вопросов:**

   Краткая история

   Маневренный планшет

   Аналитическая геометрия

   Технические требования

   Область применения

РЕФЕРАТ

Сущность настоящей дипломной работы

Аналитическая геометрия используется для установления некоторых формул в качестве контрольного или вторичного метода для решения задач на маневренном планшете, который до сих пор преподается в учебных заведениях.

Основные этапы выполненной разработки

Этап 1 - Установление начальных и конечных формул с использованием аналитической геометрии

Этап 2 - Сравнение результатов из графического решения и из формул

Этап 3 - Создание программы расчета

Этап 4 - Сравнение результатов из формул и данных из САРП

Этап 5 - Расширение проекта

Краткие сведения об объеме пояснительной записки и графической документации:

Количество листов пояснительной записки	:	88
Количество включенных иллюстраций	:	19
Количество таблиц	:	17
Количество использованных источников	:	3
Количество приложений	:	5
Количество чертежей, схем и плакатов	:	6

Ключевые слова, характеризующие содержание проекта: маневренный планшет, аналитическая геометрия, РЛС (радиолокационная станция), САРП (средство автоматической радиолокационной прокладки), предупреждение столкновений.

Объект разработки при проектировании

Использование аналитической геометрии для решения задач на маневренном планшете. Факторы движения собственного судна и других судов используются для целей данного проекта.

Цель проектирования

Некоторые формулы составляются с использованием аналитической геометрии для определения некоторых факторов движения других объектов или судов, например, CPA (дистанция кратчайшего сближения, в морских милях), $TCPA$ (время до CPA, в минутах), курс (в градусах), скорость (в узлах) и т.д.

Методы исследования

В качестве методов исследования используются математические преобразования между системами координат, преобразование формул аналитической геометрии, навигационные наблюдения и сравнения.

Полученные результаты и их новизна

Получены начальные аналитические геометрические формулы и конечные формулы, которые можно применять в вычислительной программе.

Основные технологические характеристики

Некоторые особенности РЛС/ САРП и применение вычислительной программы упомянуты для целей данного проекта.

Область применения

Применение метода аналитической геометрии для решения задач на маневренном планшете может быть использовано в учебных заведениях в качестве вторичного или контрольного метода, а также для совершенствования принципа расчета САРП.

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В настоящее время маневренный планшет до сих пор преподается в морских университетах. Это навигационное средство обеспечивает графическое решение для определения истинных и относительных факторов движения судов.

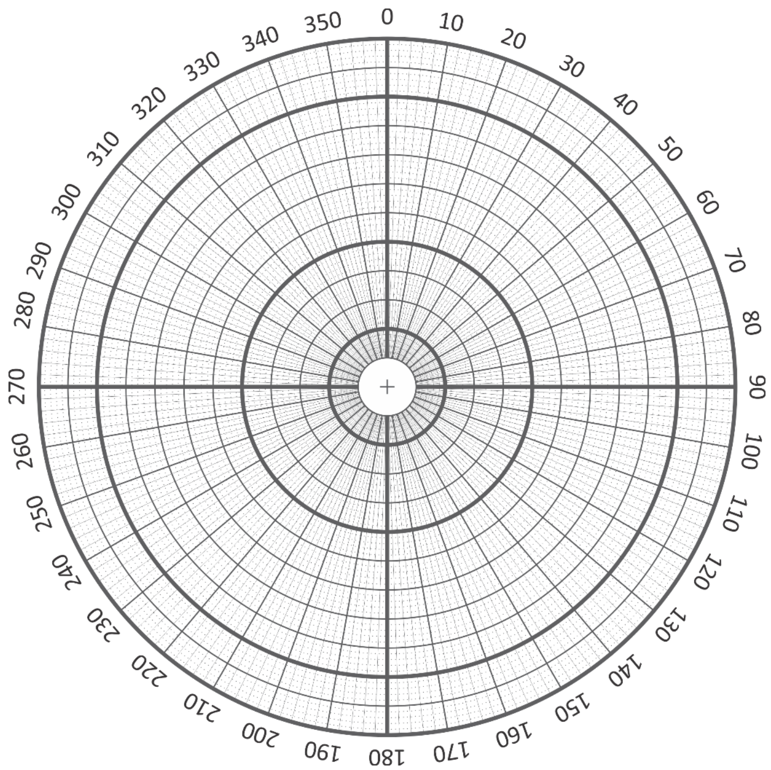


Рис. 1. Пример маневренного планшета

Проект автора ориентирован на метод решения навигационных задач в учебных заведениях.

Помимо использования графического решения в качестве начального этапа обучения, аналитическая геометрия может использоваться в качестве вторичного или контрольного метода.

В навигации навигационная полярная система координат используется для определения положения объекта с пеленгом (BRG, в градусах) и дистанцией (RNG, в морских милях). Положение может быть преобразовано в точку в декартовой системе координат, то есть с двумя осями x и y.

Некоторые формулы составляются с использованием аналитической геометрии для определения некоторых факторов движения других объектов или судов, например, CPA (дистанция кратчайшего сближения, в морских милях), $TCPA$ (время до CPA, в минутах), курс (в градусах), скорость (в узлах) и т.д.

КРАТКАЯ ИСТОРИЯ

В таблице показана краткая история РЛС и его использования:

Таблица 1. Краткая история

Время	**Событие**
В 1880-х и 1890-х годах	Были первые эксперименты многих известных физиков со всего мира, использующих радиоволны для обнаружения объектов.
Первая мировая война и Вторая мировая война	Были разработаны первые версии современных РЛС.
*	
В 1960-х годах	Первые версии САРП (*Средства автоматической радиолокационной прокладки*) были разработаны и применены в судоходной отрасли.

Период пропуска в таблице, который отмечен звездочкой, является большим вопросом для автора. В тот период маневренный планшет использовался для быстрого определения факторов движения.

Что, если метод решения автора на маневренном планшете с использованием аналитической геометрии был придуман и разработан до САРП, то есть до 1960-х годов?

Что касается будущего, то, надеемся, что метод автора может каким-то образом улучшить работу РЛС/ САРП, например, в судоходной отрасли.

ПРИМЕНЕНИЕ АНАЛИТИЧЕСКОЙ ГЕОМЕТРИИ ДЛЯ РЕШЕНИЯ ЗАДАЧ НА МАНЕВРЕННОМ ПЛАНШЕТЕ

Процесс получения начальных формул аналитической геометрии начинается с преобразования положения из навигационных полярных в декартовые координаты.

Начальные аналитические геометрические формулы для определения факторов движения преобразуются в конечные формулы, которые зависят только от входных данных, без каких-либо других промежуточных факторов.

Начальные формулы:

$$CPA = OD = \left|\overrightarrow{OD}\right| = \sqrt{{x_D}^2 + {y_D}^2}$$

$$TCPA = \frac{A_2D}{A_1A_2} \cdot \Delta t$$

$$SPD_A = 10 \cdot \sqrt{{x_2{}'}^2 + {y_2{}'}^2}$$

$$CSE_A = \left(\widehat{Oy, \overrightarrow{OA_2'}}\right) = arctan\frac{x_2'}{y_2'} = arcsin\frac{x_2'}{V_A/10} = arccos\frac{y_2'}{V_A/10}$$

Подробное объяснение приведено в основной части этого проекта.

Конечные формулы:

$$\boldsymbol{CPA = \frac{|R_1 \cdot R_2 \cdot \sin(B_1 - B_2)|}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}}$$

$$\boldsymbol{TCPA = \frac{\sqrt{{R_2}^2 - \dfrac{[R_1 \cdot R_2 \cdot \sin(B_1 - B_2)]^2}{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}} \cdot \Delta t}$$

$$\boldsymbol{SPD_A = 10 \cdot \sqrt{\begin{array}{c}\left(R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \dfrac{V}{10} \cdot \sin H\right)^2 \\ + \left(R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \dfrac{V}{10} \cdot \cos H\right)^2\end{array}}}$$

$$\boldsymbol{CSE_A = arctan\frac{R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \frac{V}{10} \cdot \sin H}{R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \frac{V}{10} \cdot \cos H}}$$

где

R – дистанция, B – пеленг, V – скорость судна, H – курс судна

ПРИМЕНЕНИЕ ВЫЧИСЛИТЕЛЬНОЙ ПРОГРАММЫ

Вычислительная программа для определения факторов движения судна (например, электронная таблица *Excel*) приведена в главе 4 п. 3, причём выходные данные вычисляются сразу после введения исходных данных.

Это еще она возможность применения языка программирования для мгновенного расчета интересующих величин.

УТВЕРЖДЕНИЯ АВТОРА

Все рисунки, представленные в этом проекте, выполнены автором с целью защиты авторских прав.

Все конечные формулы, представленные в этом проекте, составлены и изобретены автором также с целью защиты авторских прав.

Что касается языков, используемых в этом проекте:

Таблица 2. Языки

Часть	**Язык(и)**
Титульный лист, Задание, Реферат	Русский
Содержание/ Contents, Введение/ Foreword, Conclusion/ Заключение	Русский / Английский
The main part, Abbreviations, References	Английский

Британский английский используется в этом проекте для орфографии, за исключением одной фразы – «maneuvering board», которая является американским английским.

Дополнительная информация и материалы, касающиеся самого проекта, даны в Google Drive автора:

БЛАГОДАРНОСТЬ

Проект сделан со знанием и увлечением. Особая благодарность:

Морскому государственному университету имени адмирала Г.И. Невельского	*Добрым и опытным преподавателям, профессорам и консультантам* Специально Г-ну **Сливаеву Б. Г.** Г-же **Казинской О. Я.** *за добрый совет и поддержку*
WAGENBORG SHIPPING B.V.	*Компании и персоналу* Г-ну **Cees Horvers**, Г-ну **Mark Hoving**, Г-ну **Nick Bolt** и Г-же **Liz Dijk** *за любезную поддержку и организацию плавательных практик* *Добрым членам экипажа*
Фесконтракт-Интернешионал	Г-ну **Иванушкину Н. А.**

FOREWORD

ABSTRACT

Nowadays, maneuvering board is still taught in maritime institutions. This aid to navigation provides a graphical solution for determining true-motion and relative-motion factors of vessels.

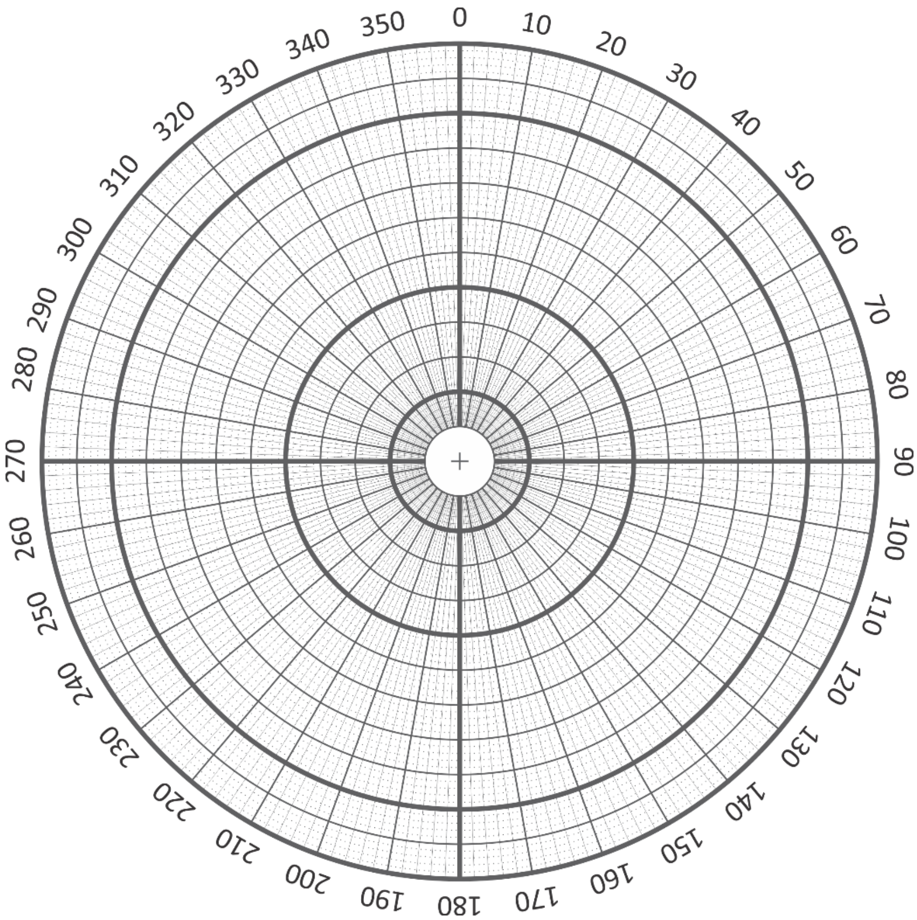


Figure 1. An example of a maneuvering board

The project of the author is focused on the method of navigation task solution at maritime universities.

Besides using graphical solution as an initial step of training, analytic geometry can be used as a secondary or control method.

In navigation, a navigational polar coordinate system is used to determine an object position with a bearing (BRG, in degrees) and a range (RNG, in nautical miles). A position can be converted into a point in a Cartesian coordinate system, i.e. with two axes x and y.

Analytic geometry is used to make up some formulas to determine those movement factors of other objects or vessels, e.g. CPA (closest point of approach, in nautical miles), $TCPA$ (time to CPA, in minutes), heading/ course (HDG, CSE, in degrees), speed (SPD, in knots), etc.

THE BRIEF HISTORY

The table below shows the brief history of the radar and its use:

Table 1. The brief history

Time	**Event**
In the 1880s and the 1890s	There were first experiments of many famous physicists from all over the world, using radio wave to detect objects.
World War I and World War II	There were the first versions of modern radar developed.
*	
In the 1960s	The first versions of ARPA (*Automatic radar plotting aid*) were developed and applied in shipping industry.

The gap period in the table, which is marked by an asterisk, is a big question to the author. In that period, maneuvering board was used to rapidly determine motion factors.

What if the author's method of solution on maneuvering board using analytic geometry had been thought up and developed before ARPA, i.e. before the 1960s?

As to the future, hopefully, the author's method can somehow improve the way radar/ ARPA works, for instance, in shipping.

APPLICATION OF ANALYTIC GEOMETRY FOR TASK SOLUTION ON MANEUVERING BOARD

The process to obtain the initial analytic geometry formulas is started with converting positions from navigational polar to Cartesian coordinates.

The initial analytic geometry formulas for determining movement factors are converted into final formulas, which depend only on input data (i.e. R, B, V and H), without any other intermediate factors.

Initial formulas:

$$CPA = OD = \left|\overrightarrow{OD}\right| = \sqrt{{x_D}^2 + {y_D}^2}$$

$$TCPA = \frac{A_2D}{A_1A_2} \cdot \Delta t$$

$$SPD_A = 10 \cdot \sqrt{{x_2'}^2 + {y_2'}^2}$$

$$CSE_A = \left(\widehat{Oy, \overrightarrow{OA_2'}}\right) = arctan\frac{x_2'}{y_2'} = arcsin\frac{x_2'}{V_A/_{10}} = arccos\frac{y_2'}{V_A/_{10}}$$

(with defined condition)

The detailed explanation is described in the main part of this project.

Final formulas:

$$\boldsymbol{CPA = \frac{|R_1 \cdot R_2 \cdot \sin(B_1 - B_2)|}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}}$$

$$\boldsymbol{TCPA = \frac{\sqrt{{R_2}^2 - \dfrac{[R_1 \cdot R_2 \cdot \sin(B_1 - B_2)]^2}{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}} \cdot \Delta t}$$

$$\boldsymbol{SPD_A = 10 \cdot \sqrt{\begin{array}{l}\left(R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \dfrac{V}{10} \cdot \sin H\right)^2 \\ + \left(R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \dfrac{V}{10} \cdot \cos H\right)^2\end{array}}}$$

$$\boldsymbol{CSE_A = arctan \frac{R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \dfrac{V}{10} \cdot \sin H}{R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \dfrac{V}{10} \cdot \cos H}}$$

(with defined condition)

where

R – range, *B* – bearing, *V* – the own vessel's speed, *H* – the own vessel's heading

IMPLEMENTATION OF A COMPUTING APPLICATION

A computing application (e.g. spreadsheet *Excel*) is applied as an example of quick determination of movement factors, with input data entered and output data received almost simultaneously.

This implementation opens another ability in application of programming language for automatic calculation.

THE AUTHOR'S STATEMENTS

All the figures presented in this project are drawn by the author for the purpose of copyright.

All the final formulas presented in this project are made up and invented by the author for the purpose of copyright, as well.

As to the languages used in this project, there are some notes as follows:

Table 2. The languages

Part	**Language(s)**
Титульный лист, Задание, Реферат	Russian
Содержание/ Contents, Введение/ Foreword, Заключение/ Conclusion	Russian/ English
The main part, Abbreviations, References	English

British English is used in this project for spelling, except only one phrase – "maneuvering board", which is American English.

More information and materials concerning the process and the project itself are shown in the author's Google Drive:

ACKNOWLEDGEMENTS

The project is made with knowledge and passion. Special thanks are due to:

Admiral Nevelskoy Maritime State University	*The kind and experienced teachers, professors and advisors* Specially Mr. **Slivaev B. G. / Сливаев Б. Г.** Ms. **Kazinskaya O. Y. / Казинская О. Я.** *for the kind advice and encouragement*
WAGENBORG SHIPPING B.V.	*The Company and The Staffs* Mr. **Cees Horvers**, Mr. **Mark Hoving**, Mr. **Nick Bolt** and Ms. **Liz Dijk** *for the kind support and the shipboard training arrangements* *The kind crewmembers of* The **MV SCHIEBORG** of Summer 2016 The **MV REGGEBORG** of Summer 2017 The **MV ANDESBORG** of Summer 2018 The **MV ARUBABORG** of Spring 2019
Fescontract-International	Mr. **Nikolai Ivanushkin** *for the kind support and the shipboard training arrangements*

1 COLREGS

Statement of Fact:

All the rules mentioned in the 1972 International Regulations for Preventing Collision at Sea (COLREGs) shall be complied at all time.

However, as to the scope of study for the diploma project, only some particular rules are focused. Those rules are references to radar and its use.

THE CONVENTION ON THE INTERNATIONAL REGULATIONS FOR PREVENTING COLLISIONS AT SEA, 1972

PART A **General**	**PART B** **Steering and sailing rules**			**PART C** **Lights and shapes**	**PART D** **Sound and light signals**	**PART E** **Exemptions**	**ANNEXES**
Rule 1 Application	***Section I - Conduct of vessels in any condition of visibility***	***Section II - Conduct of vessels in sight of one another***	***Section III - Conduct of vessels in restricted visibility***	*Rule 20* Application	*Rule 32* Definitions	*Rule 38* Application	Annex I – Positioning and technical details of lights and shapes
Rule 2 Responsibility	*Rule 4* Application	*Rule 11* Application	*Rule 19* Conduct of vessels in restricted visibility	*Rule 21* Definitions	*Rule 33* Equipment for sound signals		Annex II – Additional signals for fishing vessels fishing in close proximity
Rule 3 General definitions	*Rule 5* Look-out	*Rule 12* Sailing vessels		*Rule 22* Visibility of lights	*Rule 34* Manoeuvring and warning signals		Annex III Technical details of sound signal appliances
	Rule 6 Safe speed	*Rule 13* Overtaking		*Rule 23* Power-driven vessels underway	*Rule 35* Sound signals in restricted visibility		Annex IV – Distress signals
	Rule 7 Risk of collision	*Rule 14* Head-on situation		*Rule 24* Towing and pushing	*Rule 36* Signals to attract attention		
	Rule 8 Action to avoid collision	*Rule 15* Crossing situation		*Rule 25* Sailing vessels underway and vessels under oars	*Rule 37* Distress signals		
	Rule 9 Narrow channels	*Rule 16* Action by give-way vessel		*Rule 26* Fishing vessels			
	Rule 10 Traffic separation schemes	*Rule 17* Action by stand-on vessel		*Rule 27* Vessels not under command or restricted in their ability to manoeuvre			
		Rule 18 Responsibilities between vessels		*Rule 28* Vessels constrained by their draught			
				Rule 29 Pilot vessels			
				Rule 30 Anchored vessels and vessels aground			
				Rule 31 Seaplanes			

Figure 1.1. The COLREG contents

THE FOCUSED COLREG REGULATIONS

The summary of the focused COLREG regulations, i.e. rules 5, 6, 7, 8, 9 and 19, is shown in the sections below.

Only keywords and short phrases are used for the purpose of comprehension and memorisation of those regulations.

SECTION I - CONDUCT OF VESSELS IN ANY CONDITION OF VISIBILITY

Rule 5: Look-out

At all times

A proper look-out = sight + hearing + all available means

→ A full situation and risk appraisal

Rule 6: Safe speed

At all times

A safe speed

→ Can take proper and effective action

→ Be stopped within a distance appropriate

Factors:

(a) By all vessels:

- (i) Visibility
- (ii) Traffic density
- (iii) Vessel's manoeuvrability (stopping distance + turning ability)
- (iv) Background light at night
- (v) Wind, sea and current, and navigational hazards
- (vi) Draught + depth

(b) Additionally, by vessels with operational radar:

(i) Radar characteristics

(ii) Radar range scale

(iii) Radar detection effect

(iv) Undetected small vessels, ice and other floating objects

(v) Detected vessels’ amount, location and movement

(vi) More exact visibility assessment

***Rule 7:* Risk of collision**

(a) Use all available means → Determine risk

Doubt = Risk existence

(b) Use radar → Early risk warning

(c) Scanty information → No assumptions

(d) Considerations:

(i) Approaching vessel bearing = No change

→ Risk

(ii) Approaching a very large vessel/ a tow/ a vessel at close range

→ Risk (sometimes)

***Rule 8:* Action to avoid collision**

(a) Any action

= In accordance with the Rules + positive + in ample time + good seamanship.

(b) Alteration of course/ speed to avoid collision

= Large enough → readily apparent visually by radar

No succession of small alterations of course/ speed

(c) Sufficient sea room

→ Alteration of course alone

= The most effective action to avoid a close-quarters situation

= In good time + substantial + Not result in another close-quarters situation

(d) Action to avoid collision → Passing at a safe distance

Action effectiveness

= Carefully checked → Finally past and clear

(e) Necessary → Slacken speed/ Stop/ Reverse means of propulsion

(f)

(i) A vessel required not to impede the passage/ safe passage

→ Take early action → Allow sufficient sea room

(ii) A vessel required not to impede the passage/ safe passage approaching the other vessel so as to involve risk of collision

→ Not relieved of this obligation

(iii) A vessel the passage of which is not to be impeded

→ Fully obliged to comply with the Rules of this Part when the two vessels are approaching one another so as to involve risk of collision.

Rule 9: **Narrow channels**

(a) Keep as near as is safe and practicable

The outer limit of the channel/ fairway → Starboard side

(b) A vessel $L \leq 20\ m$ / A sailing vessel

→ Not impede the passage of a vessel which can safely navigate only within a narrow channel or fairway

(c) A vessel engaged in fishing shall

→ Not impede the passage of any other vessel navigating within a narrow channel or fairway

(d) A vessel

→ Not cross a narrow channel or fairway if such crossing impedes the passage of a vessel which can safely navigate only within such channel or fairway

The latter vessel

→ Use the sound signal prescribed in Rule 34 (d) if in doubt as to the intention of the crossing vessel

(e)

(i) Overtaking in a narrow channel or fairway

Only if the vessel to be overtaken has to take action to permit safe passing

The vessel intending to overtake

→ Sound the appropriate signal prescribed in Rule 34 (c) (i)

The vessel to be overtaken

→ In agreement = Sound the appropriate signal prescribed in Rule 34 (c) (ii) and take steps to permit safe passing

→ In doubt = Sound the signals prescribed in Rule 34 (d)

(ii) This Rule does not relieve the overtaking vessel of her obligation under Rule 13

(f) Nearing a bend or an area of a narrow channel or fairway

→ Sound the appropriate signal prescribed in Rule 34 (e)

(g) Any vessel shall

→ Avoid anchoring in a narrow channel

SECTION III - CONDUCT OF VESSELS IN RESTRICTED VISIBILITY

Rule 19: Conduct of vessels in restricted visibility

(a) Apply to vessels not in sight of one another (restricted visibility).

(b) Every vessel → Proceed at a safe speed.
A power-driven vessel → Have engines ready for immediate manoeuvre.

(c) Every vessel
→ Have due regard to the prevailing circumstances and conditions of restricted visibility when complying with the Rules of Section 1 of this Part.

(d) A vessel which detects by radar alone the presence of another vessel
→ Determine if a close-quarters situation is developing and/or risk of collision exists.
→ Shall take avoiding action in ample time, avoid:
(i) an alteration of course to port for a vessel forward of the beam, other than for a vessel being overtaken;
(ii) an alteration of course towards a vessel abeam or abaft the beam.

(e) If a risk of collision exists
→ Every vessel which hears apparently forward of the beam the fog signal of another vessel, or which cannot avoid a close-quarters situation with another vessel forward of the beam
→ Reduce speed to the minimum at which can be kept on course.
→ Take all way off and navigate with extreme caution until danger of collision is over.

2 ANALYTIC GEOMETRY

2.1 COORDINATE SYSTEMS

The coordinate systems mentioned and used in this project, i.e. *the Cartesian coordinate system*, *the polar coordinate system* and *the navigational polar coordinate system*, are described in this section.

CARTESIAN COORDINATE SYSTEM

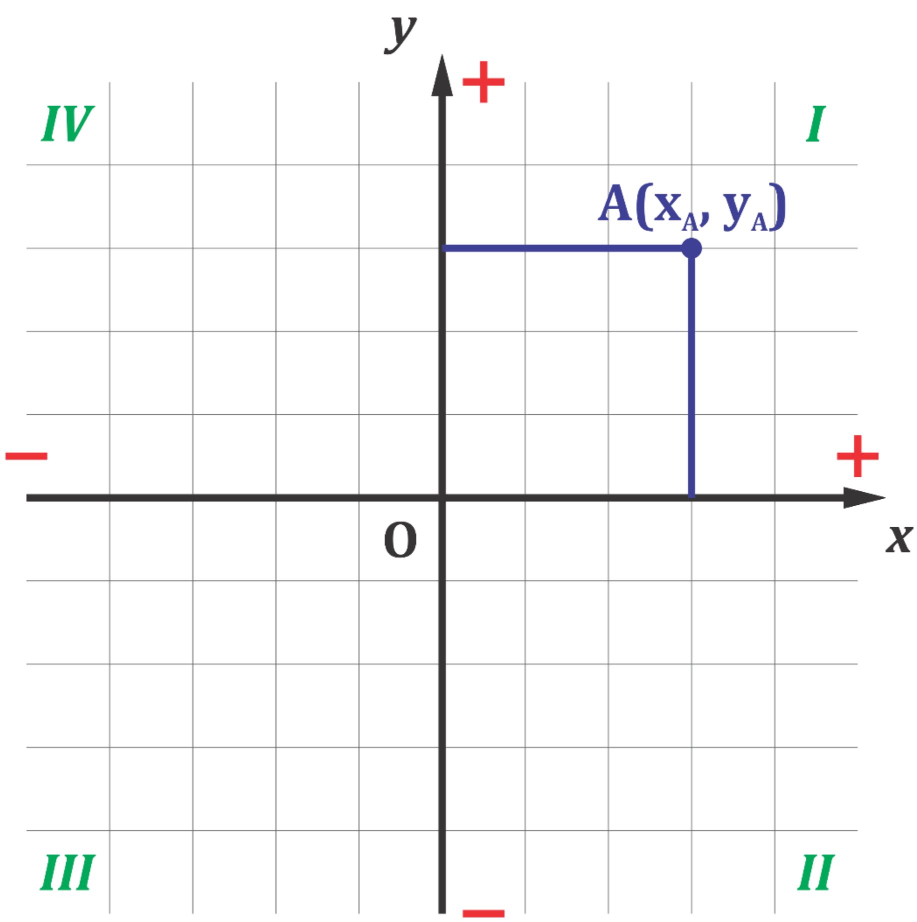


Figure 2.1.1. A Cartesian coordinate system

A position in a Cartesian coordinate system is a unique point on a plane, specified by a set of numerical coordinates, e.g. $A(x_A, y_A)$.

The main features of a Cartesian coordinate system are summarised as follows:

Table 2.1.1. The main features

Feature	**Description**
Coordinates	Numerical coordinates (x, y)
Axes	Ox – horizontal axis Oy – vertical axis
Unit	Unit of length
Origin	$O(0, 0)$
Quadrants	4 quadrants (I, II, III, IV) Direction – clockwise

The location of a point on the plane of a Cartesian coordinate system can be determined by the signs of its coordinates x and y, from which the quadrant is defined.

Table 2.1.2. The quadrants

Quadrant	***I***	***II***	***III***	***IV***
x	$x \geq 0$	$x \geq 0$	$x \leq 0$	$x \leq 0$
y	$y \geq 0$	$y \leq 0$	$y \leq 0$	$y \geq 0$
$x \cdot y$	$x \cdot y \geq 0$	$x \cdot y \leq 0$	$x \cdot y \geq 0$	$x \cdot y \leq 0$
$\frac{x}{y}$ $(y \neq 0)$	$\frac{x}{y} \geq 0$	$\frac{x}{y} \leq 0$	$\frac{x}{y} \geq 0$	$\frac{x}{y} \leq 0$

POLAR COORDINATE SYSTEM

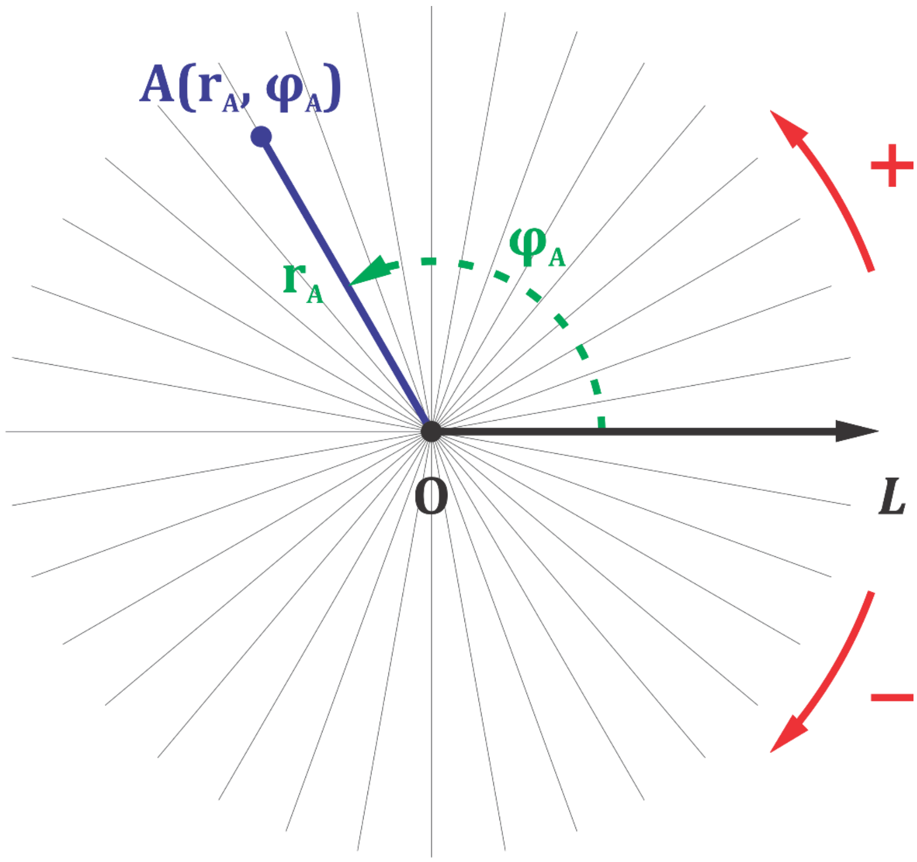


Figure 2.1.2. A polar coordinate system

A position in a polar coordinate system can be determined by a uniquely specified point on a plane, which has a defined radial distance and a certain polar angle, e.g. $A(r_A, \varphi_A)$.

In this coordinate system, attention should be drawn to the units of coordinates, as they are of different units.

The location of a point can be obtained by its angular coordinate (or azimuth), i.e. φ. There are four quadrants, I, II, III, and IV, which has the azimuth value from $0° - 90°$, $90° - 180°$, $180° - 270°$ and $270° - 360°$, respectively.

The sign of the polar angle is defined by its direction, which means, clockwise direction – negative value and anticlockwise – positive value.

In navigation, the opposite direction rule is used to determine their quadrants and signs of the polar angle. The detailed descriptions are referred in the next section.

The main features of a polar coordinate system are described as follows:

Table 2.1.3. The main features

Feature	**Description**
Coordinates	A distance from a reference point and an angle from a reference direction (r, φ) r – radial coordinate, radial distance or radius φ – angular coordinate, polar angle or azimuth
Axis	OL – polar axis
Unit	Unit of length Unit of angle
Pole	$O(0,0)$
Quadrants	4 quadrants (I, II, III, IV) Direction – anticlockwise

CONVERTING BETWEEN POLAR AND CARTESIAN COORDINATES

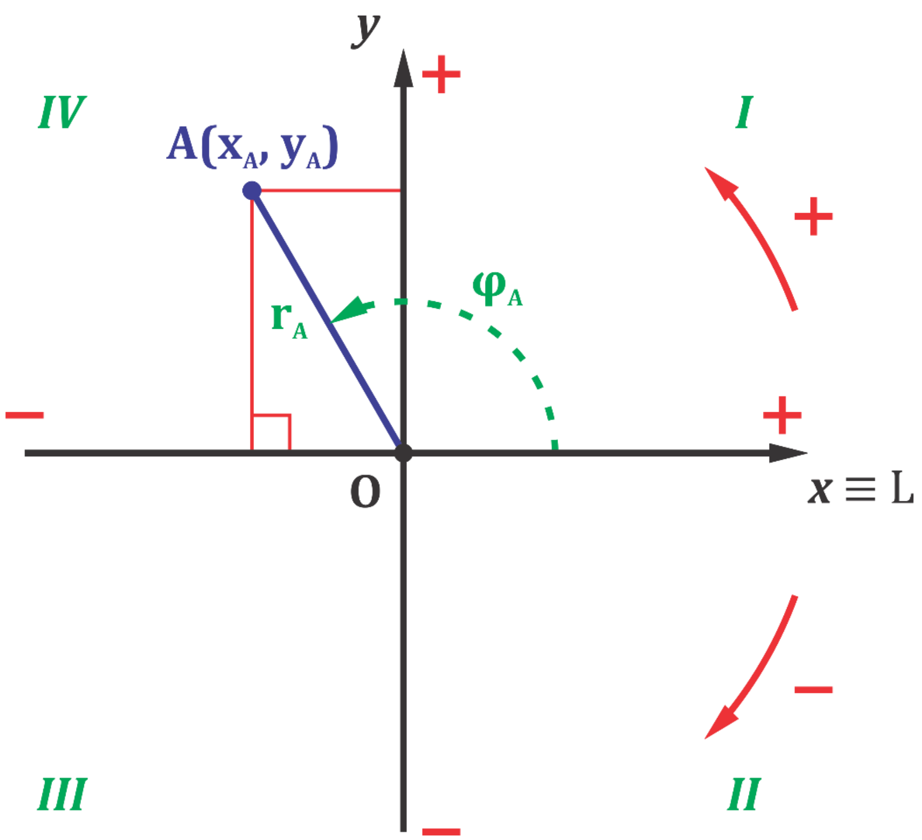


Figure 2.1.3. The relationship between polar and Cartesian coordinates

The formulas used for converting between these two coordinate systems are described as follows, in which "*the defined conditions*" mean the definition for quadrant determination:

Table 2.1.4. Conversion formulas

Polar coordinate	**Cartesian coordinate**
$r = \sqrt{x^2 + y^2}$ $\varphi = \arctan\left(\frac{y}{x}\right) = \arcsin\left(\frac{y}{r}\right) =$ $= \arccos\left(\frac{x}{r}\right)$ *(with defined conditions)*	$x = r \cdot \cos\varphi$ $y = r \cdot \sin\varphi$

NAVIGATIONAL POLAR COORDINATE SYSTEM

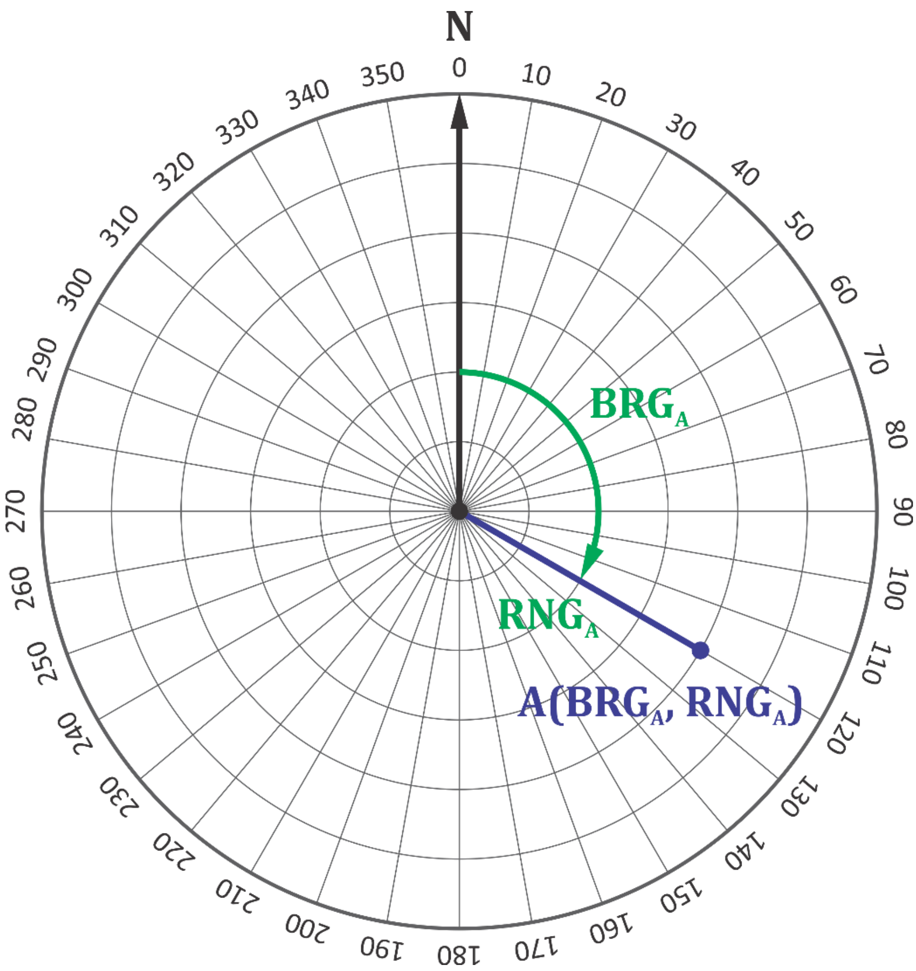


Figure 2.1.4. A navigational polar coordinate system

In navigation, a navigational polar coordinate system is used, in which the main differences from the ordinary polar coordinate system are the north axis and the direction of the bearing angle.

For the purpose of this project, the pole $O(0,0)$ represents the own vessel, and the points with alphabetical names, e.g. $A(BRG_A, RNG_A)$, show the other objects or vessels (in general, targets).

The position of a target in relation to the own vessel's position can be obtained by a bearing (BRG or B, in degrees) and a range (RNG or R, in nautical miles).

The location of a target on a plane can be determined by "*the defined conditions*", which means the quadrant the bearing belongs to.

The main features of the navigational coordinate system are shown as follows:

Table 2.1.5. The main features

Feature	**Description**
Coordinates	(BRG, RNG) or (B, R) BRG or B – bearing, RNG or R – range
Axis	ON – north axis
Unit	Degrees and nautical miles
Pole	$O(0, 0)$
Quadrants	4 quadrants (I, II, III, IV) Direction – clockwise

CONVERTING BETWEEN NAVIGATIONAL POLAR AND CARTESIAN COORDINATES

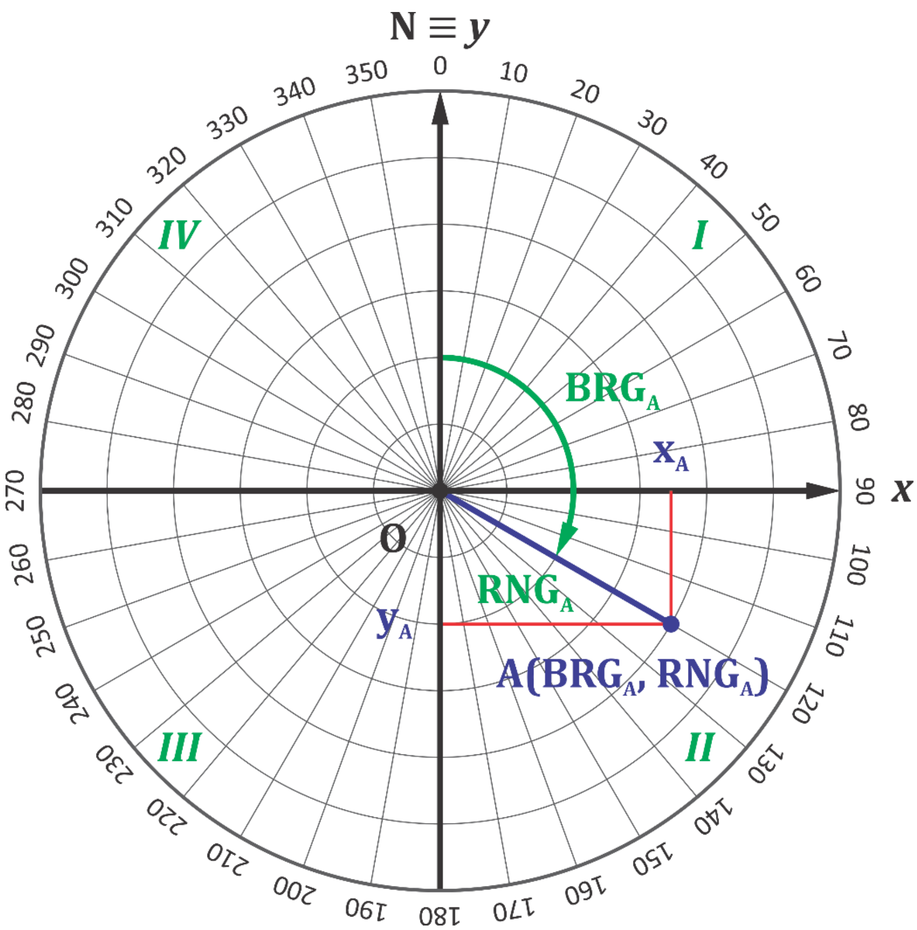


Figure 2.1.5. The relationship between navigational polar and Cartesian coordinates

As to the conversion formulas mentioned in this case, there is a change in the applied trigonometric functions sine and cosine, because of the main differences from the ordinary polar coordinate system (i.e. the north axis position and the direction of the bearing angle).

The vertical axis Oy and the north axis ON are coincident, which point to a direction of $BRG = 0°$. Consequently, the horizontal axis Ox points to a direction of $BRG = 90°$.

The concentric circles with equal intervals of distance can be used to rapidly determine the range for the purpose of navigation.

The formulas used for converting between these two coordinate systems are described as follows:

Table 2.1.6. Conversion formulas

Navigational polar coordinate	**Cartesian coordinate**
$RNG = \sqrt{x^2 + y^2}$ $BRG = \arctan\left(\frac{x}{y}\right) = \arcsin\left(\frac{x}{RNG}\right) =$ $= \arccos\left(\frac{y}{RNG}\right)$ *(with defined conditions)*	$x = RNG \cdot \sin BRG$ $y = RNG \cdot \cos BRG$

"*The defined conditions*" mean the definition for quadrant determination, which depends on the signs of the coordinates x and y:

Table 2.1.7. The defined conditions

Quadrant	$\boldsymbol{I}$	$\boldsymbol{II}$	$\boldsymbol{III}$	$\boldsymbol{IV}$
x	$x \geq 0$	$x \geq 0$	$x \leq 0$	$x \leq 0$
y	$y \geq 0$	$y < 0$	$y < 0$	$y \geq 0$
$\sin BRG$	$+$	$+$	$-$	$-$
$\cos BRG$	$+$	$-$	$-$	$+$
$\tan BRG$	$+$	$-$	$+$	$-$
$\boldsymbol{BRG}$	$\arcsin z$ $\arccos z$ $\arctan z$	$\arcsin z + 90°$ $\arccos z$ $\arctan z + 180°$	$180° - \arcsin z$ $360° - \arccos z$ $\arctan z + 180°$	$\arcsin z + 360°$ $360° - \arccos z$ $\arctan z + 360°$

2.2 LINE AND VECTOR

LINEAR EQUATION

A line in the Euclidean plane is formed as follows:

$$(\boldsymbol{l})\colon \boldsymbol{a} \cdot (\boldsymbol{x} - \boldsymbol{x_A}) + \boldsymbol{b} \cdot (\boldsymbol{y} - \boldsymbol{y_A}) = \boldsymbol{0} \tag{1}$$

where

(l) – A linear equation, a line

a, b – Coordinates of the normal vector $\vec{n} = (a, b)$ of (l)

x, y – Coordinates of any reference point $(x, y) \in (l)$

x_A, y_A – Coordinates of a point $A(x_A, y_A) \in (l)$

EUCLIDEAN VECTOR

A Euclidean vector is a line segment with a direction, connecting an initial point $A(x_A, y_A)$ with a terminal point $B(x_B, y_B)$.

$$\overrightarrow{AB} = (x_B - x_A, y_B - y_A) \tag{2}$$

where

$\overrightarrow{AB}$ – A Euclidean vector

x_A, y_A – Coordinates of an initial point $A(x_A, y_A) \in \overrightarrow{AB}$

x_B, y_B – Coordinates of a terminal point $B(x_B, y_B) \in \overrightarrow{AB}$

3 MANEUVERING BOARD

3.1 GENERAL INFORMATION

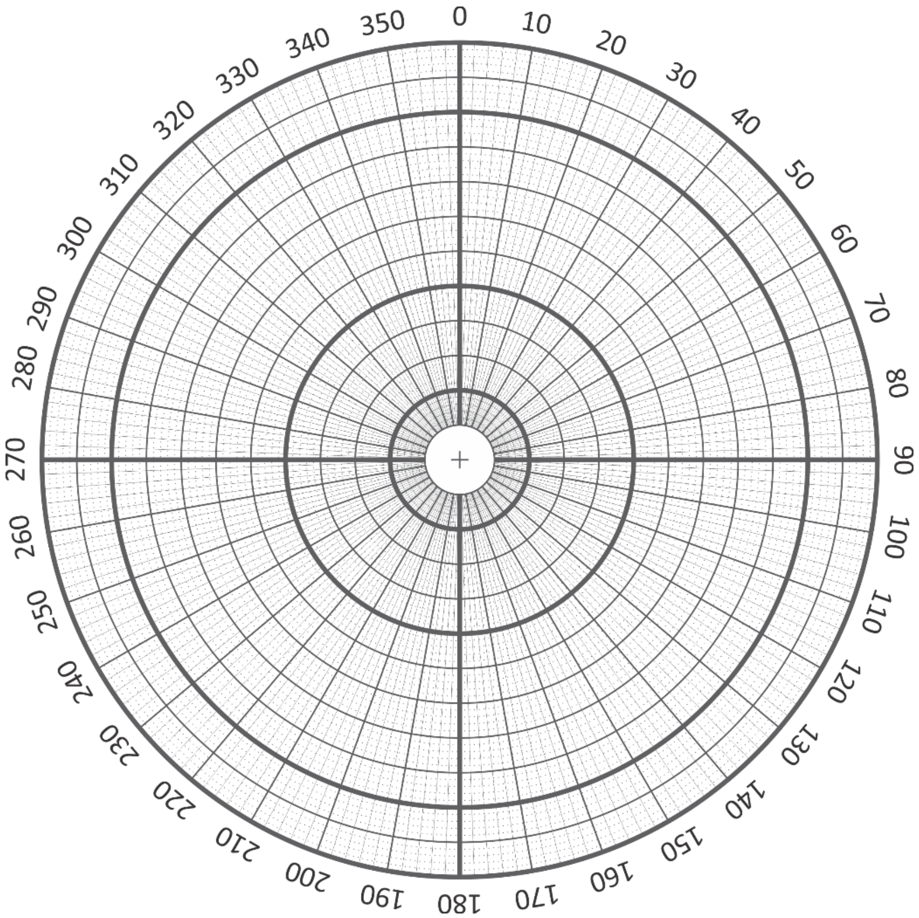


Figure 3.1.1. An example of maneuvering board

A maneuvering board, which is an aid to navigation, is used to rapidly provide movement factors of targets. A compass rose and concentric circles are printed on a writing surface (e.g. paper or board) for quick determination of targets' positions.

Dividers (i.e. a pair of compasses or a compass, drawing tool), parallel rulers and markers (or pencils) are used for manual graphical solutions on a maneuvering board.

The main features of a maneuvering board are described as follows:

Table 3.1.1. The main features

Feature	**Description**
Coordinates	(BRG, RNG) or (B, R)
Bearing	$0° - 360°$
Range	$0' - 12'$ (nautical miles)
Dangerous zone	Optional, as per requirements $0.5' - 1.0' - 2.0'$
Minimum division	Optional, as per manufactures $2°$ $0.2'$

3.2 MANUAL GRAPHICAL SOLUTION

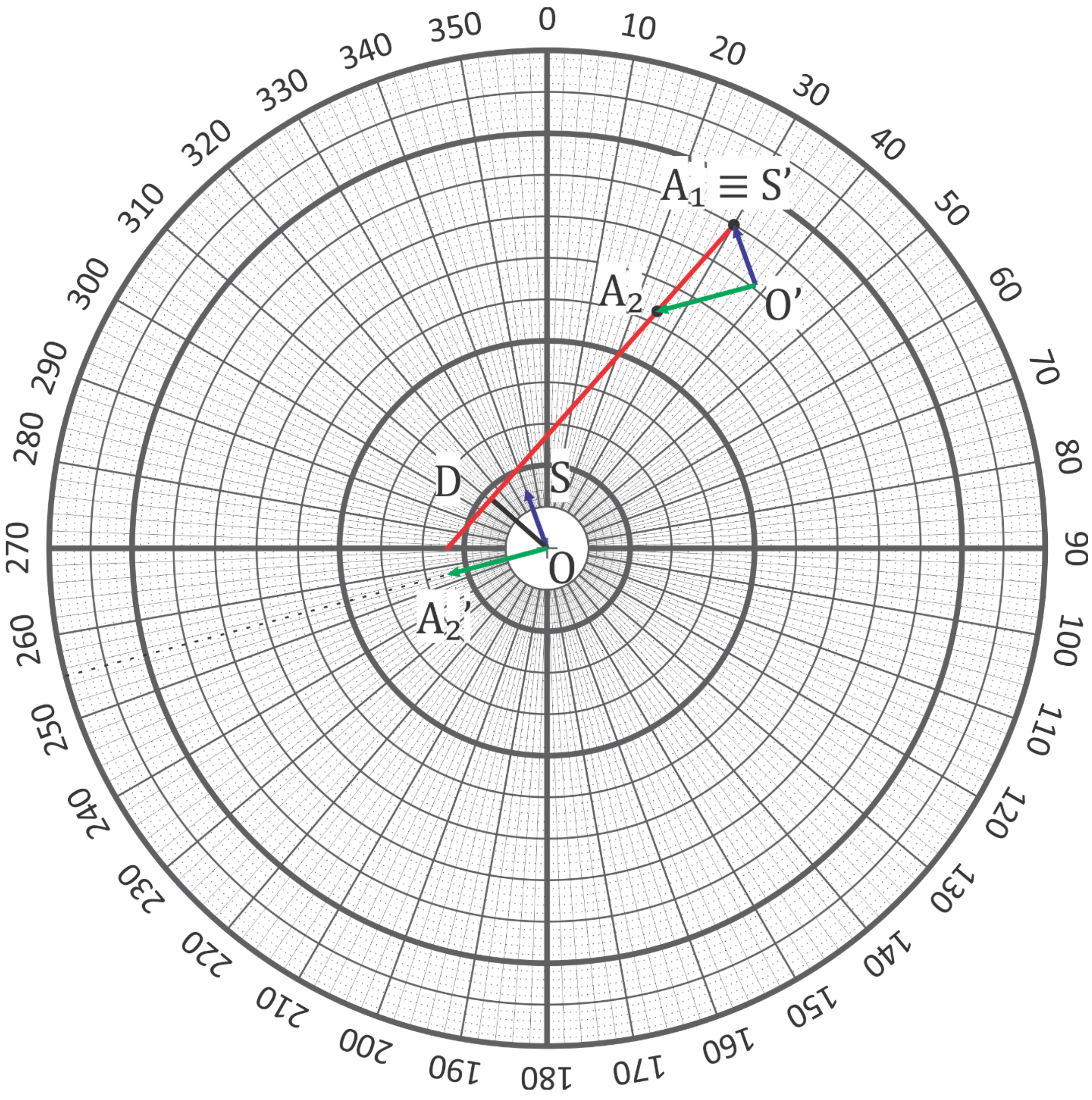


Figure 3.2.1. An example of manual graphical solution on maneuvering board

As mentioned above, for the purpose of this project, the central point $O(0,0)$ represents the own vessel, and the points with alphabetical names, e.g. $A(BRG_A, RNG_A)$, show the other objects or vessels (in general, targets).

Step 1: THE CURRENT SITUATION

The own vessel data:

For the purpose of this project, the own vessel's heading (HDG) and course (CSE) are of the same value, despite they are usually different in fact, as the own vessel's movement is always affected by outer forces (e.g. wind, wave, current, etc.).

In this case, the outer forces are considered to be absent.

$$CSE = HDG = 340°$$

$$SPD = 15\ (kn)$$

The target A data:

At moment t_1

$$\begin{cases} t_1 = 00:00:00 \\ A_1(030°, 9.0') \end{cases}$$

At moment t_2

$$\begin{cases} t_2 = 00:06:00 \\ A_2(025°, 6.3') \end{cases}$$

Interval of time between moments t_1 and t_2

$$\Delta t = t_2 - t_1 = 00:06:00 - 00:00:00 = 6\ (min)$$

Required movement factors:

$$A(CPA, TCPA, CSE_A, SPD_A) = ?$$

Step 2: PLOTTING THE OWN VESSEL VECTOR

The own vessel vector $\overrightarrow{OS}$ is drawn with direction and length as per the recent scale, i.e. the length of the vector is the distance travelled in the interval of time Δt:

$$Scale = \frac{\Delta t}{60} = \frac{6}{60} = 1:10$$

i.e.

$$\overrightarrow{OS} = (360°, 15\ kn) \longrightarrow \overrightarrow{OS} = (360°, 1.5\ nm)$$

Step 3: PLOTTING THE TARGET RELATIVE VECTOR

Two points A_1 and A_2 are plotted, then the relative movement line (RML) is drawn, which crosses through those two points towards the own vessel.

$A_1(030°, 9.0')$ and $A_2(025°, 6.3')$

Consequently, the target relative vector $\overrightarrow{A_1A_2}$ is obtained.

Step 4: DETERMINING THE TARGET $\boldsymbol{CPA}$

From the central point $O(0, 0)$, a perpendicular line toward the RML is drawn, an intersect point of which is point D.

The length of the line segment OD is determined, or simply the distance from the centre O to the RML is measured, as per the recent scale.

$$\boldsymbol{CPA = OD \approx 1.8\ (nm)}$$

Step 5: DETERMINING THE TARGET $\boldsymbol{TCPA}$

The distance travelled by the target, which is the length of the vector $\overrightarrow{A_1A_2}$, is measured, as per the recent scale.

The distance to CPA of the target, which is the length of the vector $\overrightarrow{A_2D}$, is determined, as per the recent scale, as well.

Time travelled is distance travelled divided by speed:

$$t = \frac{S}{v}$$

$$\boldsymbol{TCPA = \frac{A_2D}{A_1A_2/_{\Delta t}} = \frac{6}{2.7/_6} \approx 13.3\ (min)}$$

Step 6: DETERMINING THE TARGET TRUE VECTOR

An equivalent vector $\overrightarrow{O'S'}$ of the own vessel vector is plotted, which has the terminal point S' situated at the same position as of the initial point A_1 of the target relative vector $\overrightarrow{A_1A_2}$.

The initial point O' and the terminal point A_2 is connected, forming the target true vector $\overrightarrow{O'A_2}$.

An equivalent vector $\overrightarrow{OA_2{}'}$ is drawn, which equals to the target true vector $\overrightarrow{O'A_2}$.

$$\overrightarrow{OA_2{}'} = \overrightarrow{O'A_2}$$

i.e.

The target true vector is translated from its origin to the central point for the purpose of determination of its direction and length.

The true course of the target is defined by the compass rose, and its true speed – by the length of the vector, as per the recent scale.

$$\boldsymbol{CSE_A \approx 252.5^\circ}$$

$$\boldsymbol{SPD_A \approx 25\ kn}$$

Finally, the required movement factors of the target are determined as follows:

$$\boldsymbol{A(CPA, TCPA, CSE_A, SPD_A) = A(1.8\ nm, 13.3\ min, 252.5^\circ, 25\ kn)}$$

4 APPLICATION OF ANALYTIC GEOMETRY ON MANEUVERING BOARD

4.1 DETAILED EXPLANATION

The use of analytic geometry for task solution on maneuvering board is described by the step-by-step descriptions as below:

Step 1: THE INPUT DATA

The own vessel input data:

$$Ship[Heading\ (degrees), Speed\ (knots)] =$$

$$= S[HDG\ (°), SPD\ (kn)] =$$

$$= S(H, V)\ :\ \begin{cases} H = \overline{0°, 360°} \\ V \geq 0\ kn \end{cases}$$

The target *A* input data:

At moment t_1

$$\begin{cases} t_1 = 00{:}00{:}00 \\ A_1(BRG_1, RNG_1) = A_1(B_1, R_1) \end{cases}$$

At moment t_2

$$\begin{cases} t_2 = hh{:}mm{:}ss \\ A_2(B_2, R_2) = A_2(BRG_2, RNG_2) \end{cases}$$

Interval of time between moments t_1 and t_2

$$\Delta t = t_2 - t_1 = hh{:}mm{:}ss - 00{:}00{:}00$$

Required movement factors:

$$A(CPA, TCPA, CSE_A, SPD_A) = ?$$

Step 2: CONVERTING COORDINATES

Converting from navigational polar coordinates to Cartesian coordinates:

$$A(BRG, RNG) = A(B, R)$$

$$\sin B = \frac{x_A}{R} \Leftrightarrow x_A = R \cdot \sin B$$

$$\cos B = \frac{y_A}{R} \Leftrightarrow y_A = R \cdot \cos B$$

The Cartesian coordinates of a point A:

$$\begin{cases} x_A = R \cdot \sin B \\ y_A = R \cdot \cos B \end{cases} \quad (3)$$

These formulas are generally applied for any point on the maneuvering board.

The own vessel:

$$S\left(H, \frac{V}{10}\right) \longrightarrow S\left(\frac{V}{10} \cdot \sin H, \frac{V}{10} \cdot \cos H\right) \longrightarrow S(x_S, y_S)$$

The Cartesian coordinates of the own vessel S:

$$\begin{cases} \boldsymbol{x_S = \frac{V}{10} \cdot \sin H} \\ \boldsymbol{y_S = \frac{V}{10} \cdot \cos H} \end{cases} \tag{4}$$

The target A:

$$A_1(B_1, R_1) \longrightarrow A_1(R_1 \cdot sin\, B_1, R_1 \cdot cos\, B_1) \longrightarrow A_1(x_1, y_1)$$

$$A_2(B_2, R_2) \longrightarrow A_2(R_2 \cdot sin\, B_2, R_2 \cdot cos\, B_2) \longrightarrow A_2(x_2, y_2)$$

The Cartesian coordinates of the target A at moments t_1 and t_2:

$$\begin{cases} \boldsymbol{x_1 = R_1 \cdot \sin B_1} \\ \boldsymbol{y_1 = R_1 \cdot \cos B_1} \end{cases} \tag{5}$$

$$\begin{cases} \boldsymbol{x_2 = R_2 \cdot \sin B_2} \\ \boldsymbol{y_2 = R_2 \cdot \cos B_2} \end{cases} \tag{6}$$

Step 3: RELATIVE MOVEMENT LINE $(\boldsymbol{RML})$

Linear equation of the relative movement line $(\boldsymbol{RML})$:

$(L) \equiv (RML) \equiv (A_1A_2)$

The directional vector of (RML):

$\overrightarrow{A_1A_2} = (x_2 - x_1, y_2 - y_1) = (x_L, y_L)$

$$\begin{cases} \boldsymbol{x_L = x_2 - x_1} \\ \boldsymbol{y_L = y_2 - y_1} \end{cases} \tag{7}$$

Normal vector of (RML):

$\overrightarrow{n_L} = (y_L, -x_L)$

Points on (RML):

$A_1, A_2, D \in (L)$

Linear equation:

$$(\boldsymbol{L}): \boldsymbol{y_L \cdot (x - x_1) - x_L \cdot (y - y_1) = 0} \tag{8}$$

Step 4: $\boldsymbol{CPA}$

Point D on (RML):

$(OD) \perp (L)$

$(OD) \cap (L) = D(x_D, y_D)$

$D(x_D, y_D) \in (L)$

$\Longrightarrow y_L \cdot (x_D - x_1) - x_L \cdot (y_D - y_1) = 0$

$\Longrightarrow y_L \cdot x_D - y_L \cdot x_1 - x_L \cdot y_D + x_L \cdot y_1 = 0$

$$\Longrightarrow \boldsymbol{y_L \cdot x_D - x_L \cdot y_D = y_L \cdot x_1 - x_L \cdot y_1} \tag{9}$$

Vector $\overrightarrow{OD}$:

$\overrightarrow{OD} = (x_D, y_D)$

$\overrightarrow{OD} \perp \overrightarrow{A_1A_2}$

$\Longrightarrow \overrightarrow{OD} \cdot \overrightarrow{A_1A_2} = 0$

$$\Longrightarrow \boldsymbol{x_L \cdot x_D + y_L \cdot y_D = 0} \tag{10}$$

From (9) and (10):

$$\begin{cases} y_L \cdot x_D - x_L \cdot y_D = y_L \cdot x_1 - x_L \cdot y_1 \\ x_L \cdot x_D + y_L \cdot y_D = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} y_L \cdot x_D - x_L \cdot y_D = y_L \cdot x_1 - x_L \cdot y_1 \\ y_D = \dfrac{-x_L \cdot x_D}{y_L} \end{cases}$$

$$\Leftrightarrow \begin{cases} y_L \cdot x_D + x_L \cdot \dfrac{x_L \cdot x_D}{y_L} = y_L \cdot x_1 - x_L \cdot y_1 \\ y_D = \dfrac{-x_L \cdot x_D}{y_L} \end{cases}$$

$$\Leftrightarrow \begin{cases} x_D \cdot \left(y_L + \dfrac{{x_L}^2}{y_L} \right) = y_L \cdot x_1 - x_L \cdot y_1 \\ y_D = \dfrac{-x_L \cdot x_D}{y_L} \end{cases}$$

$$\Leftrightarrow \begin{cases} x_D = \dfrac{y_L \cdot x_1 - x_L \cdot y_1}{y_L + \dfrac{{x_L}^2}{y_L}} \\ y_D = \dfrac{-x_L \cdot x_D}{y_L} \end{cases}$$

Coordinates of the point D:

$$\begin{cases} \boldsymbol{x_D = \dfrac{y_L \cdot x_1 - x_L \cdot y_1}{y_L + \dfrac{{x_L}^2}{y_L}}} \\ \boldsymbol{y_D = \dfrac{-x_L \cdot x_D}{y_L}} \end{cases} \tag{11}$$

CPA of the target A:

Initial formula:

$$\boldsymbol{CPA = OD = \left|\overrightarrow{OD}\right| = \sqrt{x_D{}^2 + y_D{}^2}} \tag{12}$$

$$\Leftrightarrow CPA = \sqrt{\left(\frac{y_L \cdot x_1 - x_L \cdot y_1}{y_L + \frac{x_L{}^2}{y_L}}\right)^2 + \left(\frac{-x_L}{y_L} \cdot \frac{y_L \cdot x_1 - x_L \cdot y_1}{y_L + \frac{x_L{}^2}{y_L}}\right)^2}$$

$$\Leftrightarrow CPA = \left|\frac{y_L \cdot x_1 - x_L \cdot y_1}{y_L + \frac{x_L{}^2}{y_L}}\right| \cdot \sqrt{1 + \left(\frac{-x_L}{y_L}\right)^2}$$

$$\Leftrightarrow CPA = \left|\frac{y_L \cdot x_1 - x_L \cdot y_1}{y_L{}^2 + x_L{}^2} \cdot y_L\right| \cdot \sqrt{\frac{y_L{}^2 + x_L{}^2}{y_L{}^2}}$$

$$\Leftrightarrow CPA = \left|\frac{y_L \cdot x_1 - x_L \cdot y_1}{y_L{}^2 + x_L{}^2}\right| \cdot |y_L| \cdot \frac{\sqrt{y_L{}^2 + x_L{}^2}}{|y_L|}$$

$$\Leftrightarrow CPA = |y_L \cdot x_1 - x_L \cdot y_1| \cdot \frac{\sqrt{y_L{}^2 + x_L{}^2}}{y_L{}^2 + x_L{}^2}$$

$$\Leftrightarrow CPA = |y_L \cdot x_1 - x_L \cdot y_1| \cdot \frac{1}{\sqrt{y_L{}^2 + x_L{}^2}}$$

$$y_L \cdot x_1 - x_L \cdot y_1 = (y_2 - y_1) \cdot x_1 - (x_2 - x_1) \cdot y_1$$

$$\Leftrightarrow y_L \cdot x_1 - x_L \cdot y_1 = y_2 \cdot x_1 - y_1 \cdot x_1 - x_2 \cdot y_1 + x_1 \cdot y_1$$

$$\Leftrightarrow y_L \cdot x_1 - x_L \cdot y_1 = x_1 \cdot y_2 - x_2 \cdot y_1$$

$$\Leftrightarrow y_L \cdot x_1 - x_L \cdot y_1 = R_1 \cdot \sin B_1 \cdot R_2 \cdot \cos B_2 - R_2 \cdot \sin B_2 \cdot R_1 \cdot \cos B_1$$

$$\Leftrightarrow y_L \cdot x_1 - x_L \cdot y_1 = R_1 \cdot R_2 \cdot (\sin B_1 \cdot \cos B_2 - \cos B_1 \cdot \sin B_2)$$

$$\Leftrightarrow y_L \cdot x_1 - x_L \cdot y_1 = R_1 \cdot R_2 \cdot \sin(B_1 - B_2)$$

$$y_L{}^2 + x_L{}^2 = (y_2 - y_1)^2 + (x_2 - x_1)^2$$

$$\Leftrightarrow y_L{}^2 + x_L{}^2 = (R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1)^2 + (R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1)^2$$

$$= R_2{}^2 \cdot \cos B_2{}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos B_1 \cdot \cos B_2 + R_1{}^2 \cdot \cos B_1{}^2 + R_2{}^2 \cdot \sin B_2{}^2 - 2 \cdot R_1 \cdot R_2 \cdot \sin B_1 \cdot \sin B_2 + R_1{}^2 \cdot \sin B_1{}^2$$

$$= R_2{}^2 \cdot (\sin B_2{}^2 + \cos B_2{}^2) + R_1{}^2 \cdot (\sin B_1{}^2 + \cos B_1{}^2) - 2 \cdot R_1 \cdot R_2 \cdot (\cos B_1 \cdot \cos B_2 + \sin B_1 \cdot \sin B_2)$$

$$= R_1{}^2 + R_2{}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)$$

$$\Rightarrow CPA = \frac{|R_1 \cdot R_2 \cdot \sin(B_1 - B_2)|}{\sqrt{R_1{}^2 + R_2{}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}$$

Final formula:

$$\boldsymbol{CPA} = \frac{|\boldsymbol{R_1} \cdot \boldsymbol{R_2} \cdot \mathbf{sin}(\boldsymbol{B_1} - \boldsymbol{B_2})|}{\sqrt{\boldsymbol{R_1}^2 + \boldsymbol{R_2}^2 - 2 \cdot \boldsymbol{R_1} \cdot \boldsymbol{R_2} \cdot \mathbf{cos}(\boldsymbol{B_1} - \boldsymbol{B_2})}} \qquad (13)$$

Step 5: $\boldsymbol{TCPA}$

Distance to CPA of the target A:

$$\overrightarrow{A_2D} = (x_D - x_2, y_D - y_2)$$

$$A_2D = \left|\overrightarrow{A_2D}\right| = \sqrt{(x_D - x_2)^2 + (y_D - y_2)^2}$$

Distance travelled of the target A:

$$A_1A_2 = \left|\overrightarrow{A_1A_2}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$\boldsymbol{TCPA}$ of the target A:

Initial formula:

$$\boldsymbol{TCPA} = \frac{\boldsymbol{A_2D}}{\boldsymbol{A_1A_2}} \cdot \Delta \boldsymbol{t} \qquad (14)$$

$$\Longrightarrow TCPA = \frac{\sqrt{(x_D - x_2)^2 + (y_D - y_2)^2}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \cdot \Delta t$$

$$(x_D - x_2)^2 + (y_D - y_2)^2$$

$$= \left(\frac{y_L \cdot x_1 - x_L \cdot y_1}{y_L + \frac{{x_L}^2}{y_L}} - x_2 \right)^2 + \left(\frac{-x_L \cdot x_D}{y_L} - y_2 \right)^2 =$$

$$= \left(\frac{y_L \cdot x_1 - x_L \cdot y_1}{y_L + \frac{{x_L}^2}{y_L}} - x_2 \right)^2 + \left(\frac{x_L}{y_L} \cdot \frac{y_L \cdot x_1 - x_L \cdot y_1}{y_L + \frac{{x_L}^2}{y_L}} + y_2 \right)^2 =$$

$$= \left(\frac{y_L \cdot x_1 - x_L \cdot y_1}{{y_L}^2 + {x_L}^2} \cdot y_L - x_2 \right)^2 + \left(\frac{x_L}{y_L} \cdot \frac{y_L \cdot x_1 - x_L \cdot y_1}{{y_L}^2 + {x_L}^2} \cdot y_L + y_2 \right)^2 =$$

$$= \left(\frac{y_L \cdot x_1 - x_L \cdot y_1}{{y_L}^2 + {x_L}^2} \cdot y_L - x_2 \right)^2 + \left(\frac{y_L \cdot x_1 - x_L \cdot y_1}{{y_L}^2 + {x_L}^2} \cdot x_L + y_2 \right)^2 =$$

$$= \left(\frac{y_L \cdot x_1 - x_L \cdot y_1}{{y_L}^2 + {x_L}^2} \right)^2 \cdot ({y_L}^2 + {x_L}^2) + {x_2}^2 + {y_2}^2 - 2 \cdot \frac{y_L \cdot x_1 - x_L \cdot y_1}{{y_L}^2 + {x_L}^2} \cdot (y_L \cdot x_2 - x_L \cdot y_2) =$$

$$= \frac{(y_L \cdot x_1 - x_L \cdot y_1)^2}{{y_L}^2 + {x_L}^2} + {x_2}^2 + {y_2}^2 - 2 \cdot \frac{y_L \cdot x_1 - x_L \cdot y_1}{{y_L}^2 + {x_L}^2} \cdot (y_L \cdot x_2 - x_L \cdot y_2)$$

$$= {x_2}^2 + {y_2}^2 - \frac{(y_L \cdot x_1 - x_L \cdot y_1)^2}{{y_L}^2 + {x_L}^2} \cdot \left(2 \cdot \frac{y_L \cdot x_2 - x_L \cdot y_2}{y_L \cdot x_1 - x_L \cdot y_1} - 1 \right)$$

$$y_L \cdot x_1 - x_L \cdot y_1 = R_1 \cdot R_2 \cdot \sin(B_1 - B_2)$$

$${y_L}^2 + {x_L}^2 = {R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)$$

$${x_2}^2 + {y_2}^2 = {R_2}^2 \cdot \sin {B_2}^2 + {R_2}^2 \cdot \cos {B_2}^2 = {R_2}^2 \cdot (\sin {B_2}^2 + \cos {B_2}^2) = {R_2}^2$$

$$y_L \cdot x_2 - x_L \cdot y_2 = (y_2 - y_1) \cdot x_2 - (x_2 - x_1) \cdot y_2$$

$$\Leftrightarrow y_L \cdot x_2 - x_L \cdot y_2 = y_2 \cdot x_2 - y_1 \cdot x_2 - x_2 \cdot y_2 + x_1 \cdot y_2$$

$$\Leftrightarrow y_L \cdot x_2 - x_L \cdot y_2 = x_1 \cdot y_2 - x_2 \cdot y_1$$

$$\Leftrightarrow y_L \cdot x_2 - x_L \cdot y_2 = y_L \cdot x_1 - x_L \cdot y_1$$

$$\Leftrightarrow y_L \cdot x_2 - x_L \cdot y_2 = R_1 \cdot R_2 \cdot \sin(B_1 - B_2)$$

$$\Longrightarrow 2 \cdot \frac{y_L \cdot x_2 - x_L \cdot y_2}{y_L \cdot x_1 - x_L \cdot y_1} - 1 = 2 \cdot \frac{R_1 \cdot R_2 \cdot \sin(B_1 - B_2)}{R_1 \cdot R_2 \cdot \sin(B_1 - B_2)} - 1 = 2 - 1 = 1$$

$$\Longrightarrow (x_D - x_2)^2 + (y_D - y_2)^2 = {x_2}^2 + {y_2}^2 - \frac{(y_L \cdot x_1 - x_L \cdot y_1)^2}{{y_L}^2 + {x_L}^2}$$

$$\Longrightarrow (x_D - x_2)^2 + (y_D - y_2)^2 = {R_2}^2 - \frac{[R_1 \cdot R_2 \cdot \sin(B_1 - B_2)]^2}{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}$$

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = {y_L}^2 + {x_L}^2$$

$$\Leftrightarrow (x_2 - x_1)^2 + (y_2 - y_1)^2 = {R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)$$

$$\Longrightarrow TCPA = \frac{\sqrt{{R_2}^2 - \frac{[R_1 \cdot R_2 \cdot \sin(B_1 - B_2)]^2}{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}} \cdot \Delta t$$

Final formula:

$$\boldsymbol{TCPA} = \frac{\sqrt{\boldsymbol{R_2}^2 - \frac{[\boldsymbol{R_1} \cdot \boldsymbol{R_2} \cdot \mathbf{sin}(\boldsymbol{B_1} - \boldsymbol{B_2})]^2}{\boldsymbol{R_1}^2 + \boldsymbol{R_2}^2 - \mathbf{2} \cdot \boldsymbol{R_1} \cdot \boldsymbol{R_2} \cdot \mathbf{cos}(\boldsymbol{B_1} - \boldsymbol{B_2})}}}{\sqrt{\boldsymbol{R_1}^2 + \boldsymbol{R_2}^2 - \mathbf{2} \cdot \boldsymbol{R_1} \cdot \boldsymbol{R_2} \cdot \mathbf{cos}(\boldsymbol{B_1} - \boldsymbol{B_2})}} \cdot \Delta \boldsymbol{t} \quad (15)$$

Step 6: SPEED OF THE TARGET $\boldsymbol{A}$

The equivalent vector of the target A relative speed vector:

$${A_2}'({x_2}', {y_2}') : \overrightarrow{S{A_2}'} = \overrightarrow{A_1 A_2}$$

$$\overrightarrow{S{A_2}'} = ({x_2}' - x_S, {y_2}' - y_S)$$

$$\overrightarrow{A_1 A_2} = (x_2 - x_1, y_2 - y_1)$$

$$\Longrightarrow \begin{cases} {x_2}' - x_S = x_2 - x_1 \\ {y_2}' - y_S = y_2 - y_1 \end{cases}$$

$$\Longleftrightarrow \begin{cases} {x_2}' = x_2 - x_1 + x_S \\ {y_2}' = y_2 - y_1 + y_S \end{cases} \quad (16)$$

The equivalent vector of the target A true speed vector:

$$\overrightarrow{OA_2{}'} = (x_2{}', y_2{}')$$

The value of the target A true speed is determined as per the recent scale:

$$Scale = \frac{\Delta t}{60} = \frac{6}{60} = 1:10$$

$$\Longrightarrow \frac{SPD_A}{10} = OA_2{}' = \left|\overrightarrow{OA_2{}'}\right| = \sqrt{x_2{}'^2 + y_2{}'^2}$$

The true speed of the target $\boldsymbol{A}$:

Initial formula:

$$\boldsymbol{SPD_A = 10 \cdot \sqrt{x_2{}'^2 + y_2{}'^2}} \tag{17}$$

$$\Leftrightarrow SPD_A = 10 \cdot \sqrt{(x_2 - x_1 + x_S)^2 + (y_2 - y_1 + y_S)^2}$$

$$x_2 - x_1 + x_S = R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \frac{V}{10} \cdot \sin H$$

$$y_2 - y_1 + y_S = R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \frac{V}{10} \cdot \cos H$$

$$\Longrightarrow SPD_A = 10 \cdot \sqrt{\begin{array}{c}\left(R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \frac{V}{10} \cdot \sin H\right)^2 \\ + \left(R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \frac{V}{10} \cdot \cos H\right)^2\end{array}}$$

Final formula:

$$\boldsymbol{SPD_A = 10 \cdot \sqrt{\begin{array}{c}\left(R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \frac{V}{10} \cdot \sin H\right)^2 \\ + \left(R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \frac{V}{10} \cdot \cos H\right)^2\end{array}}} \qquad (18)$$

Step 7: COURSE OF THE TARGET *A*

The true course of the target A:

Initial formula:

$$\boldsymbol{CSE_A = \left(\widehat{Oy, \overrightarrow{OA_2'}}\right) = arctan \frac{x_2'}{y_2'} = arcsin \frac{x_2'}{V_A/10} = arccos \frac{y_2'}{V_A/10}} \qquad (19)$$

For the purpose of this project, the arctangent function is applied for calculation of the target A true course:

$$CSE_A = arctan \frac{x_2'}{y_2'}$$

$$\Leftrightarrow CSE_A = arctan\frac{R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \frac{V}{10} \cdot \sin H}{R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \frac{V}{10} \cdot \cos H}$$

Final formula:

$$\boldsymbol{CSE_A = arctan\frac{R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \frac{V}{10} \cdot \sin H}{R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \frac{V}{10} \cdot \cos H}} \tag{20}$$

"*The defined conditions*" for determination of the bearing quadrant are described as follows:

Table 4.1.1. The defined conditions

Quadrant	$\boldsymbol{I}$	$\boldsymbol{II}$	$\boldsymbol{III}$	$\boldsymbol{IV}$
x	$x \geq 0$	$x \geq 0$	$x \leq 0$	$x \leq 0$
y	$y \geq 0$	$y < 0$	$y < 0$	$y \geq 0$
$\tan BRG$	$+$	$-$	$+$	$-$
$\boldsymbol{BRG}$	$\arctan z$	$\arctan z + 180°$	$\arctan z + 180°$	$\arctan z + 360°$

where

$z = \tan BRG$

4.2 COMPARISON: ANALYTIC GEOMETRY METHOD VS MANUAL GRAPHICAL SOLUTION

The comparison between the author's method of solution using analytic geometry formulas and the manual graphical solution is described as below:

Step 1: THE INPUT DATA

The own vessel input data:

$S(340°, 15\ kn)$

The target *A* input data:

At moment t_1

$$\begin{cases} t_1 = 00:00:00 \\ A_1(030°, 9.0') \end{cases}$$

At moment t_2

$$\begin{cases} t_2 = 00:06:00 \\ A_2(025°, 6.3') \end{cases}$$

Interval of time between moments

$\Delta t = t_2 - t_1 = 6\ min$

Step 2: CONVERTING COORDINATES

Converting from navigational polar coordinate to Cartesian coordinate:

The own vessel:

$$S\left(340°, \frac{15}{10}\right) \longrightarrow S\left(\frac{15}{10} \cdot \sin 340°, \frac{15}{10} \cdot \cos 340°\right)$$

$$\longrightarrow S(x_S \approx -0.513, y_S \approx 1.410)$$

The target A:

$$A_1(030°, 9.0') \to A_1(9.0 \cdot \sin 30°, 9.0 \cdot \cos 30°)$$

$$\to A_1(x_1 \approx 4.500, y_1 \approx 7.794)$$

$$A_2(025°, 6.3') \to A_2(6.3 \cdot \sin 25°, 6.3 \cdot \cos 25°)$$

$$\to A_2(x_2 \approx 2.662, y_2 \approx 5.710)$$

Step 3: RELATIVE MOVEMENT LINE (RML)

Linear equation of the relative movement line (RML):

$$(L) \equiv (RML) \equiv (A_1A_2)$$

Directional vector of (RML):

$$\overrightarrow{A_1A_2} = (x_2 - x_1, y_2 - y_1) = (2.662 - 4.500, 5.710 - 7.794)$$

$$= (x_L \approx -1.838, y_L \approx -2.084)$$

Step 4: $\boldsymbol{CPA}$

Coordinates of the point D:

$$x_D = \frac{y_L \cdot x_1 - x_L \cdot y_1}{y_L + \frac{{x_L}^2}{y_L}}$$

$$\Longrightarrow x_D = \frac{-2.084 \cdot 4.500 - (-1.838) \cdot 7.794}{-2.084 + \frac{(-1.838)^2}{-2.084}} \approx -1.335$$

$$y_D = \frac{-x_L \cdot x_D}{y_L}$$

$$\Longrightarrow y_D = \frac{-(-1.838) \cdot (-1.335)}{-2.084} \approx 1.177$$

$\boldsymbol{CPA}$ of the target $\boldsymbol{A}$:

Initial formula:

$$CPA = OD = \left|\overrightarrow{OD}\right| = \sqrt{{x_D}^2 + {y_D}^2}$$

$$\Longrightarrow CPA = \sqrt{(-1.335)^2 + (1.177)^2} \approx 1.780\ (nm)$$

Final formula:

$$CPA = \frac{|R_1 \cdot R_2 \cdot \sin(B_1 - B_2)|}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}$$

$$\Longrightarrow CPA = \frac{|9.0 \cdot 6.3 \cdot \sin(030 - 025)|}{\sqrt{9.0^2 + 6.3^2 - 2 \cdot 9.0 \cdot 6.3 \cdot \cos(030 - 025)}}$$

$$\Longrightarrow CPA \approx 1.778392557\ (nm)$$

Step 5: $\boldsymbol{TCPA}$

Distance to CPA of the target A:

$$A_2D = \left|\overrightarrow{A_2D}\right| = \sqrt{(x_D - x_2)^2 + (y_D - y_2)^2}$$

$$= \sqrt{(-1.335 - 2.662)^2 + (1.177 - 5.710)^2} \approx 6.044\ (nm)$$

Distance travelled of the target A:

$$A_1A_2 = \left|\overrightarrow{A_1A_2}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-1.838)^2 + (-2.084)^2} \approx 2.779$$

$TCPA$ of the target A:

Initial formula:

$$TCPA = \frac{A_2D}{A_1A_2} \cdot \Delta t$$

$$\Longrightarrow TCPA = \frac{6.044}{2.779} \cdot 6 \approx 13.049\ (min)$$

Final formula:

$$TCPA = \frac{\sqrt{{R_2}^2 - \frac{[R_1 \cdot R_2 \cdot \sin(B_1 - B_2)]^2}{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}} \cdot \Delta t$$

$$\Longrightarrow TCPA = \frac{\sqrt{6.3^2 - \frac{[9.0 \cdot 6.3 \cdot \sin(030 - 025)]^2}{9.0^2 + 6.3^2 - 2 \cdot 9.0 \cdot 6.3 \cdot \cos(030 - 025)}}}{\sqrt{9.0^2 + 6.3^2 - 2 \cdot 9.0 \cdot 6.3 \cdot \cos(030 - 025)}} \cdot 6$$

$$\Longrightarrow TCPA \approx 13.049946140\ (min)$$

Step 6: SPEED OF THE TARGET *A*

$$\begin{cases} {x_2}' = x_2 - x_1 + x_S = 2.662 - 4.500 + (-0.513) = -2.351 \\ {y_2}' = y_2 - y_1 + y_S = 5.710 - 7.794 + 1.410 = -0.674 \end{cases}$$

True speed of the target A:

Initial formula:

$$SPD_A = 10 \cdot \sqrt{{x_2}'^2 + {y_2}'^2}$$

$$\Longrightarrow SPD_A = 10 \cdot \sqrt{(-2.351)^2 + (-0.674)^2} \approx 24.457\ (kn)$$

Final formula:

$$SPD_A = 10 \cdot \sqrt{\begin{gathered}\left(R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \frac{V}{10} \cdot \sin H\right)^2 \\ + \left(R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \frac{V}{10} \cdot \cos H\right)^2\end{gathered}}$$

$$\Longrightarrow SPD_A = 10 \cdot \sqrt{\begin{gathered}\left(6.3 \cdot \sin 025 - 9.0 \cdot \sin 030 + \frac{15}{10} \cdot \sin 340\right)^2 \\ + \left(6.3 \cdot \cos 025 - 9.0 \cdot \cos 030 + \frac{15}{10} \cdot \cos 340\right)^2\end{gathered}}$$

$$\Longrightarrow SPD_A \approx 24.455212000\ (kn)$$

Step 7: COURSE OF THE TARGET A

The defined conditions:

In order to determine which quadrant, the bearing belongs to, "*the defined conditions*" are applied as per initial formula and final formula, as well.

As per initial formula:

$$\begin{cases} x_2{}' = x_2 - x_1 + x_S = 2.662 - 4.500 + (-0.513) = -2.351 < 0 \\ y_2{}' = y_2 - y_1 + y_S = 5.710 - 7.794 + 1.410 = -0.674 < 0 \end{cases}$$

$$\begin{cases} x_2{}' = -2.351 < 0 \\ y_2{}' = -0.674 < 0 \end{cases}$$

$$\Longrightarrow Quadrant\ III$$

As per final formula:

$$\begin{cases} x_2{}' = R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \dfrac{V}{10} \cdot \sin H \\ y_2{}' = R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \dfrac{V}{10} \cdot \cos H \end{cases}$$

$$\begin{cases} x_2{}' = 6.3 \cdot \sin 025 - 9.0 \cdot \sin 030 + \dfrac{15}{10} \cdot \sin 340 \approx -2.350535166 < 0 \\ y_2{}' = 6.3 \cdot \cos 025 - 9.0 \cdot \cos 030 + \dfrac{15}{10} \cdot \cos 340 \approx -0.674950645 < 0 \end{cases}$$

$$\Longrightarrow Quadrant\ III$$

Table 4.2.1. The defined conditions

Quadrant	***I***	***II***	***III***	***IV***
x	$x \geq 0$	$x \geq 0$	$x \leq 0$	$x \leq 0$
y	$y \geq 0$	$y < 0$	$y < 0$	$y \geq 0$
$\boldsymbol{BRG}$	$\arctan z$	$\arctan z + 180°$	$\arctan z + 180°$	$\arctan z + 360°$

True course of the target A:

Initial formula:

$$CSE_A = \left(\widehat{Oy, \overrightarrow{OA_2{}'}}\right) = arctan\frac{x_2{}'}{y_2{}'} = arcsin\frac{x_2{}'}{V_A/_{10}} = arccos\frac{y_2{}'}{V_A/_{10}}$$

$$\Longrightarrow CSE_A = arctan\frac{-2.351}{-0.674} + 180° \approx 254.003°$$

Final formula:

$$CSE_A = arctan\frac{R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \frac{V}{10} \cdot \sin H}{R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \frac{V}{10} \cdot \cos H}$$

$$\Longrightarrow CSE_A = arctan\frac{6.3 \cdot \sin 025 - 9.0 \cdot \sin 030 + \frac{15}{10} \cdot \sin 340}{6.3 \cdot \cos 025 - 9.0 \cdot \cos 030 + \frac{15}{10} \cdot \cos 340} + 180°$$

$$\Longrightarrow CSE_A = arctan\frac{-2.350535166}{-0.674950645} + 180°$$

$$\Longrightarrow CSE_A \approx 253.978705400°$$

COMPARISON

The results from manual graphical solution, from both initial and final sets of formulas are compared for an assessment of the author method's accuracy.

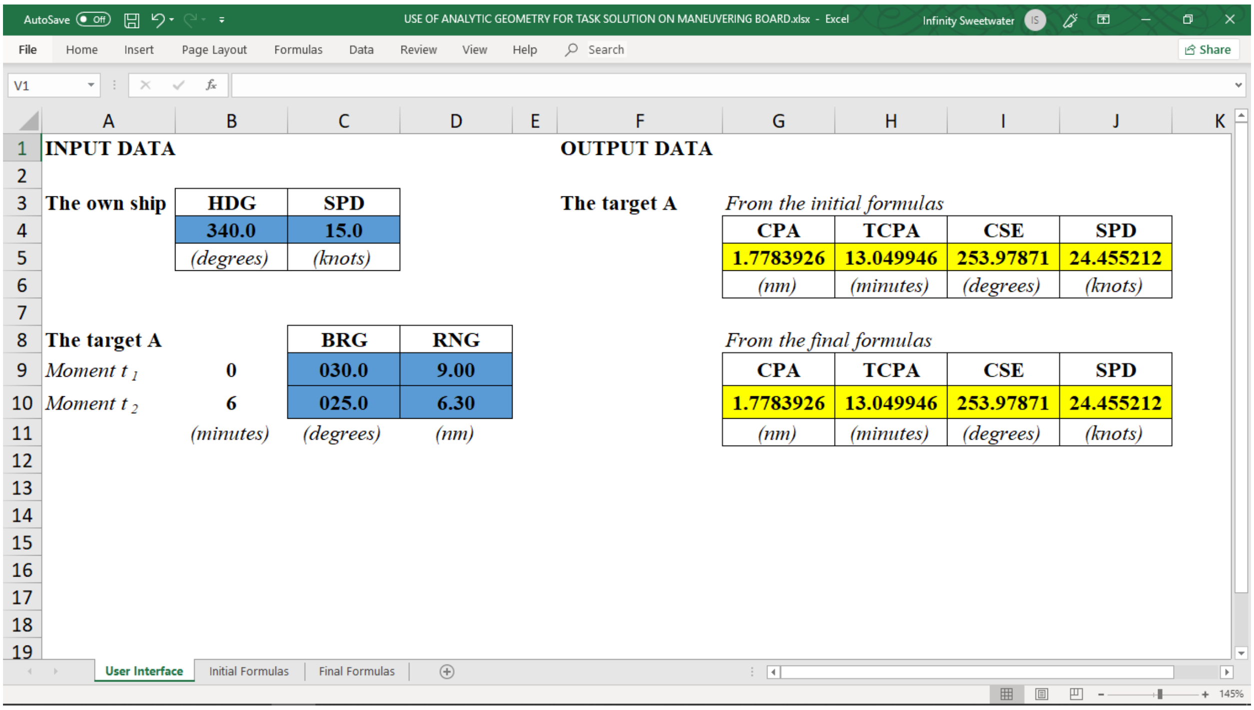
Table 4.2.2. The comparison

Value	**Manual graphical solution**	**Analytic geometry Initial formulas**	**Analytic geometry Final formulas**
CPA	$1.8\ (nm)$	$1.780\ (nm)$	$1.778392557\ (nm)$
$TCPA$	$13.3\ (min)$	$13.049\ (min)$	$13.049946140\ (min)$
CSE	$252.5°$	$254.003°$	$253.978705400°$
SPD	$25\ (kn)$	$24.457\ (kn)$	$24.455212000\ (kn)$

4.3 IMPLEMENTATION OF A COMPUTING APPLICATION

The author’s method with both initial and final sets of formulas is applied in a computing program, in this case, the spreadsheet *Microsoft Excel*.

There are three worksheets (tabs) in the provided *Excel* file, which are: “*User Interface*”, “*Initial Formulas*” and “*Final Formulas*”.



	A	B	C	D	E	F	G	H	I	J
1	**INPUT DATA**					**OUTPUT DATA**				
2										
3	**The own ship**	**HDG**	**SPD**			**The target A**	*From the initial formulas*			
4		**340.0**	**15.0**				**CPA**	**TCPA**	**CSE**	**SPD**
5		*(degrees)*	*(knots)*				**1.7783926**	**13.049946**	**253.97871**	**24.455212**
6							*(nm)*	*(minutes)*	*(degrees)*	*(knots)*
7										
8	**The target A**		**BRG**	**RNG**			*From the final formulas*			
9	*Moment t_1*	**0**	**030.0**	**9.00**			**CPA**	**TCPA**	**CSE**	**SPD**
10	*Moment t_2*	**6**	**025.0**	**6.30**			**1.7783926**	**13.049946**	**253.97871**	**24.455212**
11		*(minutes)*	*(degrees)*	*(nm)*			*(nm)*	*(minutes)*	*(degrees)*	*(knots)*

Figure 4.3.1. The worksheet “User Interface”

More information concerning the Excel workbook is presented in the author’s Google Drive:

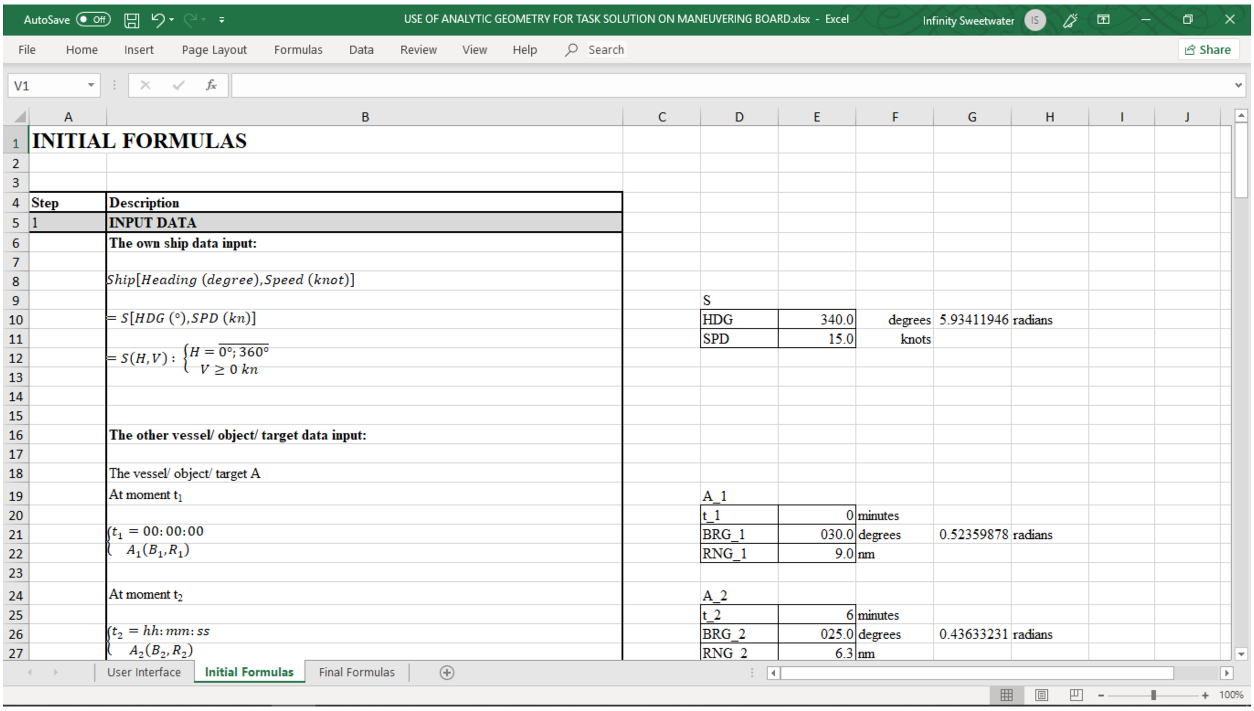


Figure 4.3.2. The worksheet "Initial Formulas"

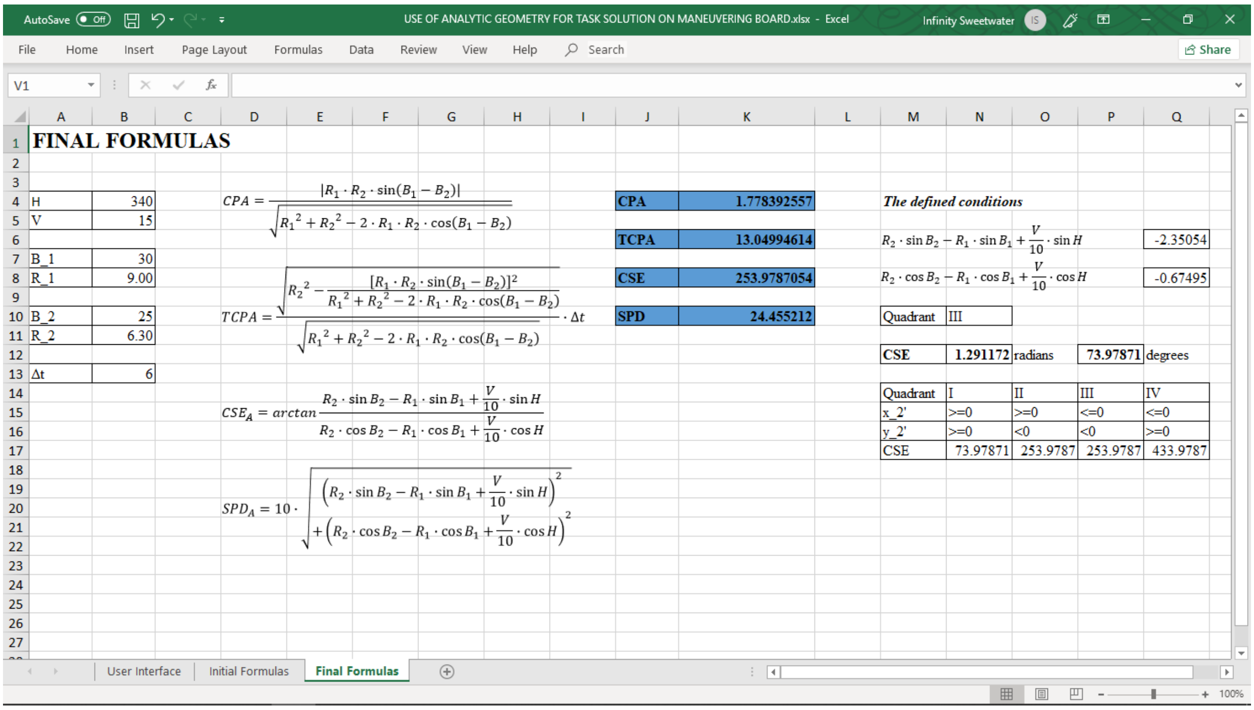


Figure 4.3.3. The worksheet "Final Formulas"

The results from the initial and final formulas are compared for the purpose of assessment of the author's method accuracy.

4.4 COMPARISON: ANALYTIC GEOMETRY METHOD VS ARPA DATA

THE SITUATIONS

Navigational observations using radar, visual lookouts and all available means were taken place on board the *MV Arubaborg* during the author's shipboard training in the spring, 2019.

Figure 4.4.1. One of the observed targets

The situations of observations were different, e.g. underway, at anchor, approaching target and passed target.

The radar screen displays, radar data and ARPA data are observed, recorded, compared, analysed and calculated for the purpose of this project.

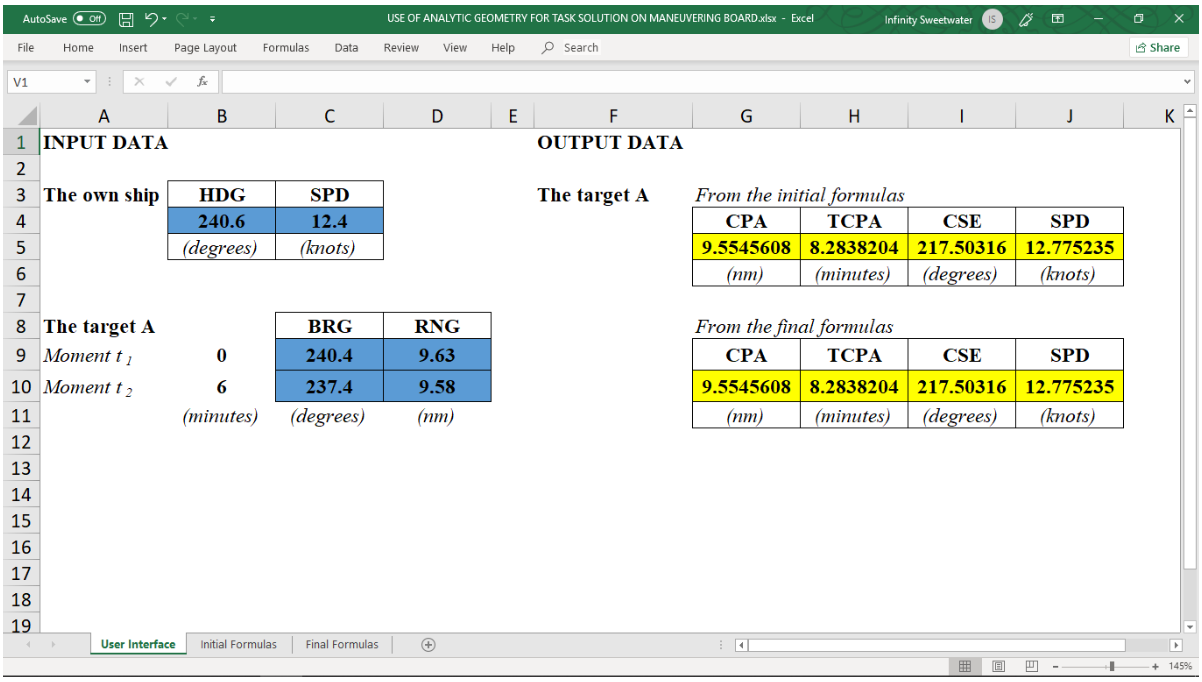
The record data are shown as follows:

Table 4.4.1. The record data

Record data	**Observation 1**	**Observation 2**	**Observation 3**	**Observation 4**
Date	02.04.2019	27.04.2019	27.04.2019	27.04.2019
Position	$44°\ 17.9'\ N$ $024°\ 07.3'\ W$	$28°\ 53.4'\ N$ $089°\ 20.6'\ W$	$28°\ 53.4'\ N$ $089°\ 20.6'\ W$	$28°\ 53.4'\ N$ $089°\ 20.6'\ W$
Situation	Underway	At anchor	At anchor	At anchor
Own vessel	$S(240.6°, 12.4\ kn)$	$S(109.5°, 0.01\ kn)$	$S(103.7°, 0.02\ kn)$	$S(101.0°, 0.06\ kn)$
Target *A*	$t_1 = 12:24:00$ $A_1(240.4°, 9.63')$ $t_2 = 12:30:00$ $A_2(237.4°, 9.58')$	$t_1 = 14:36:00$ $A_1(244.7°, 0.93')$ $t_2 = 14:42:00$ $A_2(188.0°, 0.19')$	$t_1 = 14:43:00$ $A_1(141.9°, 0.20')$ $t_2 = 14:49:00$ $A_2(084.1°, 0.96')$	$t_1 = 14:53:00$ $A_1(275.7°, 0.65')$ $t_2 = 14:59:00$ $A_2(252.3°, 1.42')$
ARPA data	$CPA = 9.56\ nm$ $TCPA = 8.1\ min$ $CSE = 217.4°$ $SPD = 12.9\ kn$	$CPA = 0.18\ nm$ $TCPA = 0.5\ min$ $CSE = 076.4°$ $SPD = 8.43\ kn$	$CPA = 0.18\ nm$ $TCPA = -6.63\ min$ $CSE = 073.7°$ $SPD = 8.49\ kn$	$CPA = 0.36\ nm$ $TCPA = -9.65\ min$ $CSE = 237.8°$ $SPD = 8.51\ kn$

More images and videos concerning the sequences of navigational observations are shown in the author's Google Drive:

OUTPUT DATA FROM THE COMPUTING APPLICATION APPLIED ANALYTIC GEOMETRY



INPUT DATA

The own ship	HDG	SPD
	240.6	12.4
	(degrees)	(knots)

The target A		BRG	RNG
Moment t_1	0	240.4	9.63
Moment t_2	6	237.4	9.58
	(minutes)	(degrees)	(nm)

OUTPUT DATA

The target A

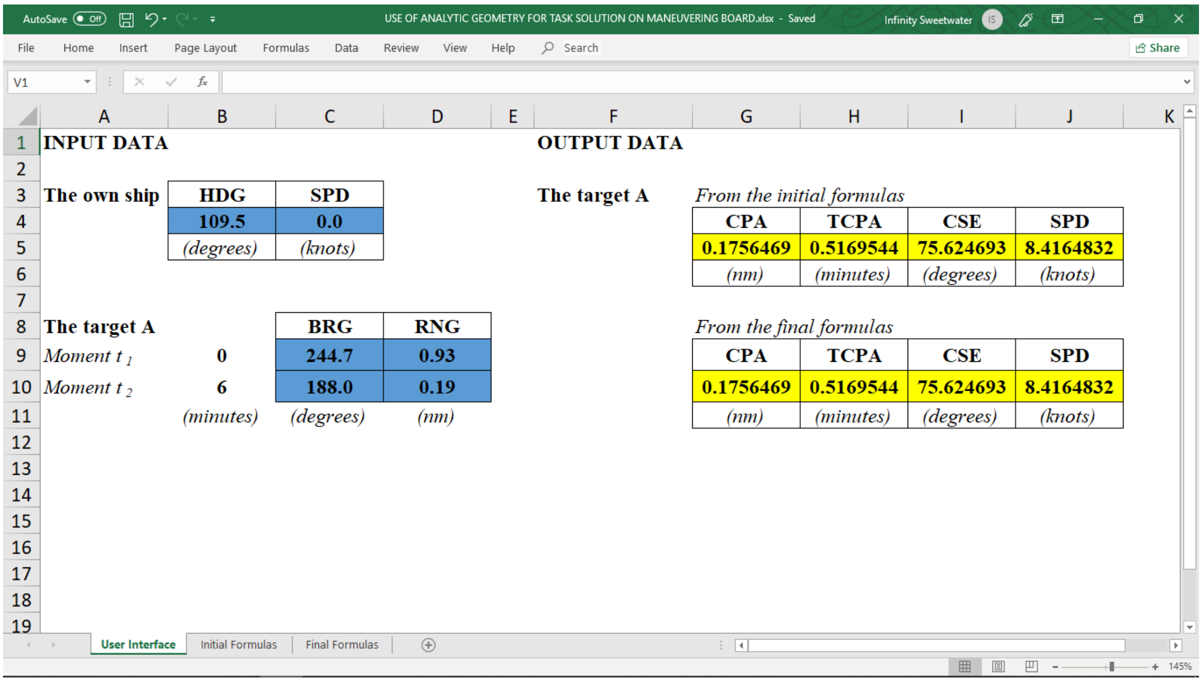
From the initial formulas

CPA	TCPA	CSE	SPD
9.5545608	8.2838204	217.50316	12.775235
(nm)	(minutes)	(degrees)	(knots)

From the final formulas

CPA	TCPA	CSE	SPD
9.5545608	8.2838204	217.50316	12.775235
(nm)	(minutes)	(degrees)	(knots)

Figure 4.4.2. Output data of Observation 1



INPUT DATA

The own ship	HDG	SPD
	109.5	0.0
	(degrees)	(knots)

The target A		BRG	RNG
Moment t_1	0	244.7	0.93
Moment t_2	6	188.0	0.19
	(minutes)	(degrees)	(nm)

OUTPUT DATA

The target A

From the initial formulas

CPA	TCPA	CSE	SPD
0.1756469	0.5169544	75.624693	8.4164832
(nm)	(minutes)	(degrees)	(knots)

From the final formulas

CPA	TCPA	CSE	SPD
0.1756469	0.5169544	75.624693	8.4164832
(nm)	(minutes)	(degrees)	(knots)

Figure 4.4.3. Output data of Observation 2

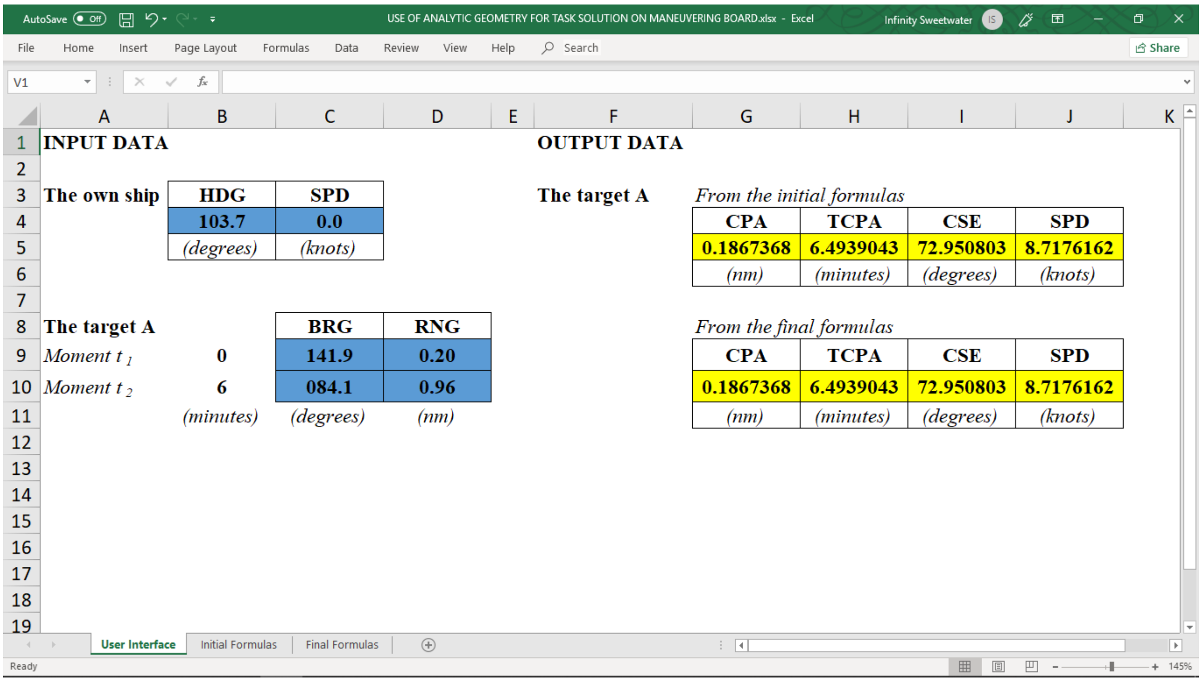


Figure 4.4.4. Output data of Observation 3

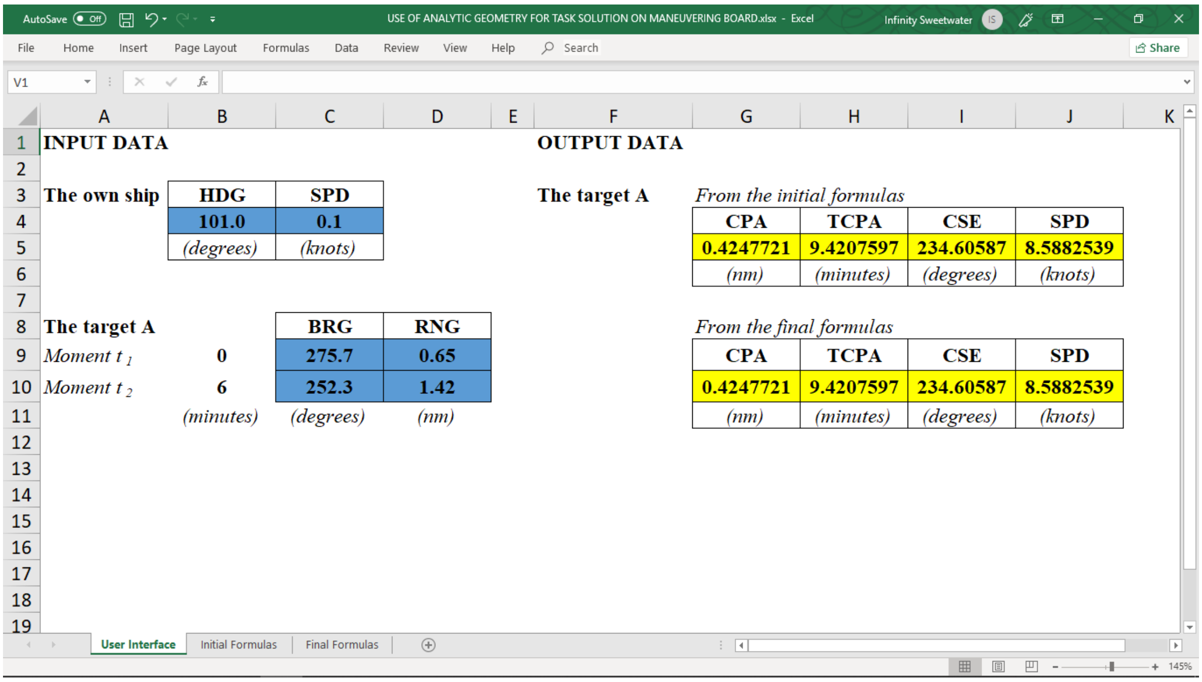


Figure 4.4.5. Output data of Observation 4

COMPARISON

The ARPA data and the output data from the calculating program applied analytic geometry are compared as below:

Table 4.4.2. The comparison

Comparison data	**ARPA data**	**Analytic geometry method**
Observation 1	$CPA = 9.56\ (nm)$ $TCPA = 8.1\ (min)$ $CSE = 217.4°$ $SPD = 12.9\ (kn)$	$CPA = 9.555\ (nm)$ $TCPA = 8.284\ (min)$ $CSE = 217.503°$ $SPD = 12.775\ (kn)$
Observation 2	$CPA = 0.18\ (nm)$ $TCPA = 0.5\ (min)$ $CSE = 076.4°$ $SPD = 8.43\ (kn)$	$CPA = 0.176\ (nm)$ $TCPA = 0.517\ (min)$ $CSE = 075.625°$ $SPD = 8.416\ (kn)$
Observation 3	$CPA = 0.18\ (nm)$ $\boldsymbol{TCPA = -6.63\ (min)}$ $CSE = 073.7°$ $SPD = 8.49\ (kn)$	$CPA = 0.187\ (nm)$ $\boldsymbol{TCPA = 6.494\ (min)}$ $CSE = 072.951°$ $SPD = 8.718\ (kn)$
Observation 4	$CPA = 0.36\ (nm)$ $\boldsymbol{TCPA = -9.65\ (min)}$ $CSE = 237.8°$ $SPD = 8.51\ (kn)$	$CPA = 0.425\ (nm)$ $\boldsymbol{TCPA = 9.421\ (min)}$ $CSE = 234.606°$ $SPD = 8.588\ (kn)$

The $TCPA$ **negative values** from ARPA data indicate that the targets have passed the CPA, which are necessary to take into account by the author's analytic geometry method for further improvement and development.

5 IMPROVEMENT AND EXTENSION

The author's project is going to be developed, extended and improved as it is still in the initial stage itself.

There are some aspects, which can be taken into account during the future improvement and development:

1. Calculation of **leeway angle** and **drift angle** (See *Figure 5.1.*)
2. The effects from **wind** and **current**
3. Manoeuvring **prediction** and **suggestion**
4. **Collision avoidance** with many targets
5. Calculation of $\boldsymbol{BCT}$ (bow crossing time) and $\boldsymbol{BCR}$ (bow crossing range)

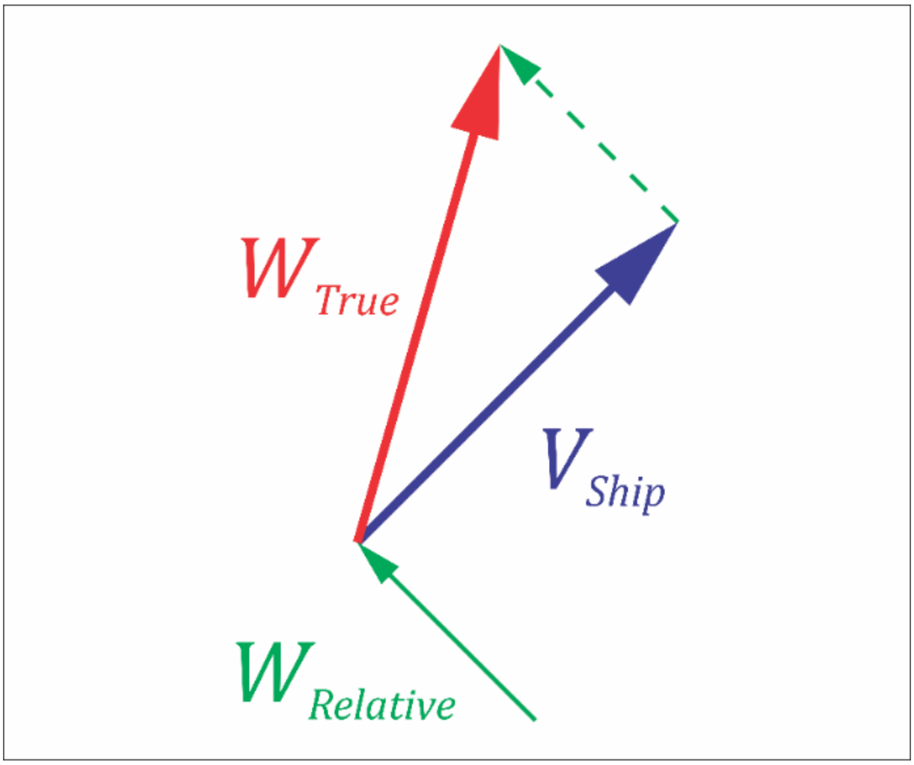


Figure 5.1. True wind vector triangle

As to the implementation of the author's method for the future, it can be suggested that, together with AI (*Artificial Intelligence*), "**a manoeuvring advisor**" can be developed. The AI, with all input data of the own vessel and the vicinity information (i.e. meteorological condition, traffic density, etc.), can rapidly calculate and display the suggested manoeuvres to the navigator. The aftermath of the suggested manoeuvres can also be predicted and presented.

It is completely obvious that, the computer and algorithm can comply with the regulations (e.g. COLREG), more precisely, with very quick suggested solutions.

CONCLUSION

It is pleased to state that the author's thesis is going in the right direction with some initial stage's significant results. This project has proved that the author's solution is **a precise method** to determine vessel's movement factors.

On the one hand, there is a manual solution, using graphical method on a maneuvering board, in which the time consumption, skills and experience are required.

On the other hand, the author's method comes with only formulas, which can be implemented in any computing application, or programming algorithm.

$$CPA = \frac{|R_1 \cdot R_2 \cdot \sin(B_1 - B_2)|}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}$$

$$TCPA = \frac{\sqrt{{R_2}^2 - \frac{[R_1 \cdot R_2 \cdot \sin(B_1 - B_2)]^2}{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}} \cdot \Delta t$$

$$SPD_A = 10 \cdot \sqrt{\left(R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \frac{V}{10} \cdot \sin H\right)^2 + \left(R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \frac{V}{10} \cdot \cos H\right)^2}$$

$$CSE_A = arctan \frac{R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \frac{V}{10} \cdot \sin H}{R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \frac{V}{10} \cdot \cos H}$$

The author's method for determining the vessel's movement factors, applied in this project, is creative and understandable. To obtain the desired results, it is enough to take bearings and distances to targets, and to have them entered into the program, which greatly facilitate the work of an officer of the watch (OOW), who must make the right decision in a short period of time.

The main advantages of the author's method with final formulas applied analytic geometry are listed as follows:

1. The final formulas depend only on the input data, i.e. the target's bearings, ranges, and the own vessel's heading/ course, speed, which can significantly improve the speed and accuracy of calculation, as well as the way ARPA calculates its data.
2. The scale is optional, which can be of any interval of time between moments, e.g. 6 sec, 10 sec, 1 min, 2 min, 3 min, etc.
3. The results do not depend on the human factors.
4. Besides manual graphical solution, the author's method can be built up as a secondary method or a control method.
5. *What if the author's method of solution on maneuvering board using analytic geometry had been thought up and developed before ARPA, i.e. before the 1960s?*

It is an honour and a pride for the author that the thesis has been successful proved, as per initial stage, from the slightest idea, which suddenly appeared in the author's mind two years ago (i.e. in the spring semester, 2017), during a class period of the "Maneuvering board" course, after many observations and calculations, up to now.

Special thanks and best wishes are being expressed by the author.

ЗАКЛЮЧЕНИЕ

Приятно заявить, что тезис автора идет в правильном направлении с некоторыми значительными результатами на начальном этапе. Этот проект доказал, что метод решения автора является точным методом определения факторов движения судна.

С одной стороны, существует решение с использованием графического метода на маневренном планшете, в котором требуются затраты времени, навыки и опыт. С другой стороны, метод автора поставляется только с формулами, которые могут быть применены в любом вычислительной программе или алгоритме программирования.

$$CPA = \frac{|R_1 \cdot R_2 \cdot \sin(B_1 - B_2)|}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}$$

$$TCPA = \frac{\sqrt{{R_2}^2 - \dfrac{[R_1 \cdot R_2 \cdot \sin(B_1 - B_2)]^2}{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}} \cdot \Delta t$$

$$SPD_A = 10 \cdot \sqrt{\begin{array}{l}\left(R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \dfrac{V}{10} \cdot \sin H\right)^2 \\ + \left(R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \dfrac{V}{10} \cdot \cos H\right)^2\end{array}}$$

$$CSE_A = arctan \frac{R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \dfrac{V}{10} \cdot \sin H}{R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \dfrac{V}{10} \cdot \cos H}$$

Метод автора для определения факторов движения судна, применяемый в данном проекте, является творческим и понятным. Для получения нужных результатов достаточно иметь пеленги и дистанции целей и ввести их в программу, что значительно облегчит работу штурмана, который должен принимать правильное решение в короткий промежуток времени.

Основные преимущества метода автора с конечными формулами аналитической геометрии перечислены следующим образом:

1. Конечные формулы зависят только от входных данных, то есть от пеленгов, дальностей до целей и курса, скорости собственного судна, что может значительно повысить скорость и точность расчета, а также улучшить способ расчета данных САРП.
2. Масштаб является произвольным, который может иметь любое значение интервала времени между моментами, например, $6\,sec$, $10\,sec$, $1\,min$, $2\,min$, $3\,min$, и т.д.
3. Результаты не зависят от человеческого фактора.
4. Помимо графического решения, метод автора может быть построен как вторичный метод или метод контроля.
5. *Что, если метод решения автора на маневренном планшете с использованием аналитической геометрии был придуман и разработан до САРП, то есть до 1960-х годов?*

Для автора большая честь и гордость за то, что этот тезис был успешно доказан на начальном этапе, из малейшей идеи, которая внезапно возникла в голове автора два года назад (т.е. в весеннем семестре 2017 года), во время периода занятий курса «Маневренного планшета», после многих наблюдений и расчетов, до сих пор.

Особая благодарность и наилучшие пожелания выражаются автором.

ABBREVIATIONS
СОКРАЩЕНИЯ

A

ARPA	Automatic Radar Plotting Aid

B

B	bearing	BRG	bearing

C

COLREG	The Convention on the International Regulations for Preventing Collision at Sea, 1972	CPA	closest point of approach
CSE	course		

D

D	distance

E

ECDIS	Electronic Chart Display and Information System	e.g.	exempli gratia, for example

F

(spare)

G

(spare)

H

HDG	heading

I

i.e.	id est, that is, in other words

J

(spare)

K

kn	knots (nautical miles per hour)

L

L	line, relative movement line

M

min	minutes

N

N	North	nm	nautical mile

O

O	origin of a coordinate system	OOW	officer of the watch

P

(spare)

Q

(spare)

R

R	range	RNG	range

S

S	the own ship	sec	seconds
SPD	speed		

T

t_i	a moment of time

U

(spare)

V

V	speed

W

W	wind speed vector

X

x	horizontal axis, horizontal coordinate

Y

y	vertical axis, vertical coordinate

Z

z	a number (any)

REFERENCES

СПИСОК ИСПОЛЬЗОВАННЫХ ИСТОЧНИКОВ

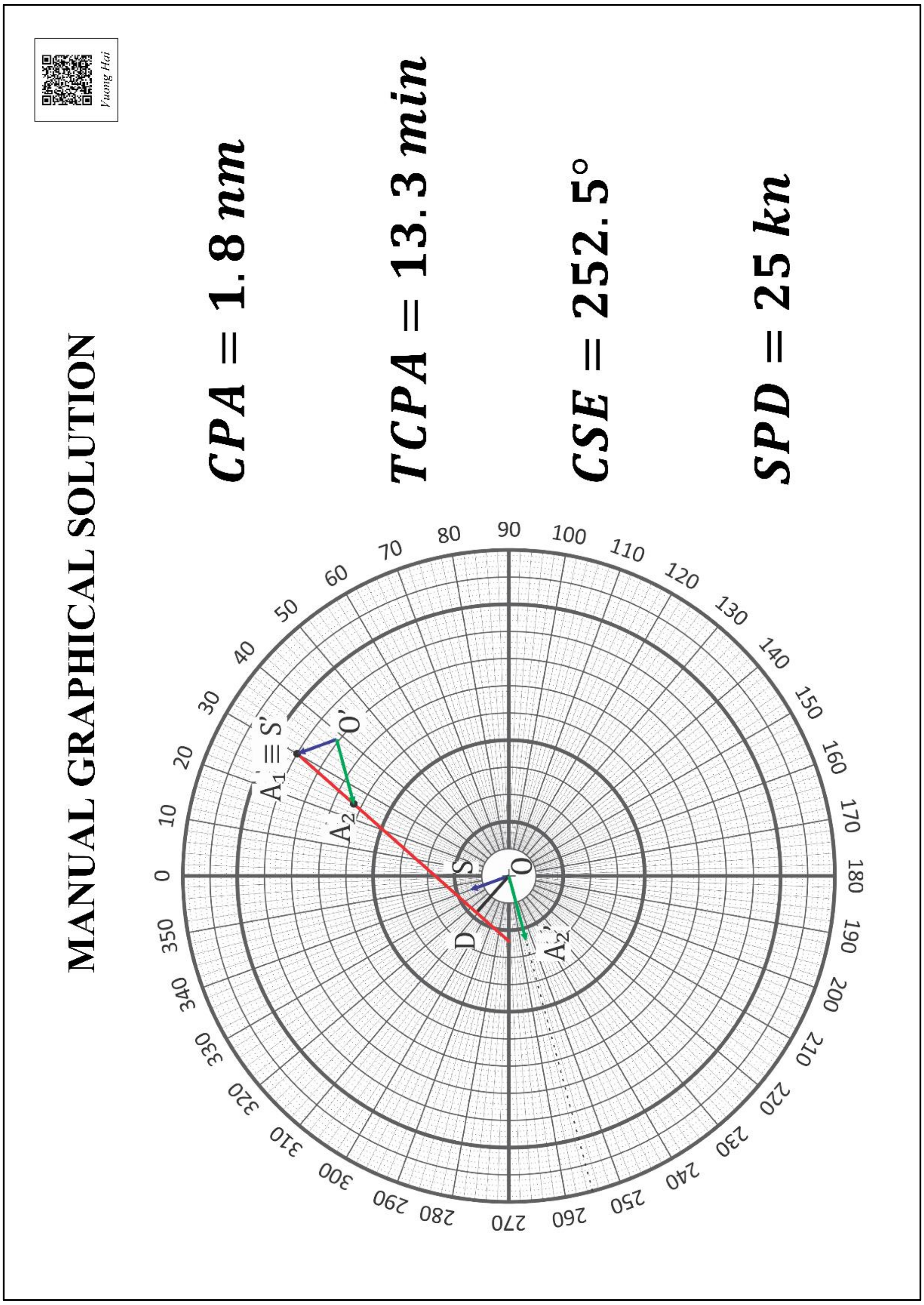
[1] International Maritime Organization (IMO), Part B – Steering and Sailing, Section 1 – Conduct of vessels in any condition of visibility (Rules 5, 6, 7, 8, 9), Section III – Conduct of vessels in restricted visibility (Rule 19), The International Regulations for Preventing Collisions at Sea, 1972 (COLREGs), 1972.

[2] Maritime Safety Information Center, National Imagery and Mapping Agency, Chapter 6 - Maneuvering board manual, Pub 1310. Radar Navigation and Maneuvering Board Manual, 2001, Page 248.

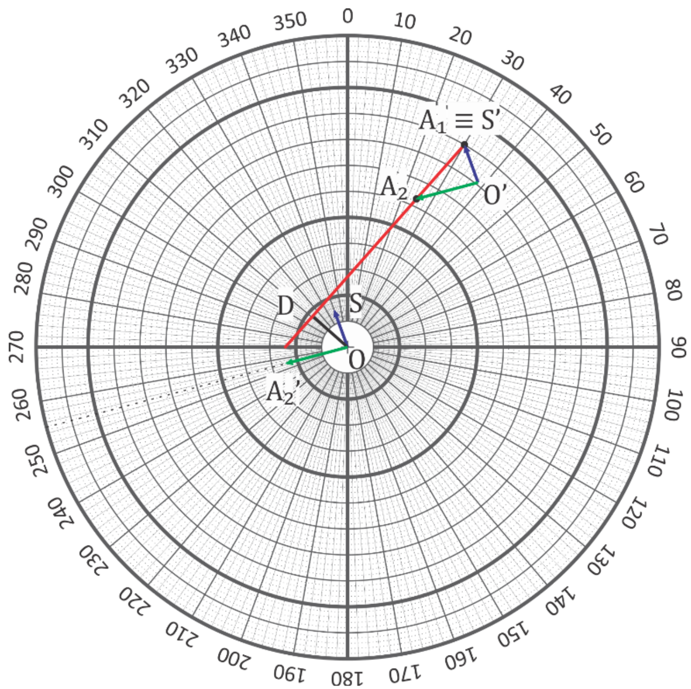
[3] Vuong Hai, The author’s Google Drive, “The 2019 Project”, 2019, Access link: *https://drive.google.com/open?id=1-5V4FMk4teyqaPVifbLAu-nJSHMRZ06K*.

ANNEXES
ПРИЛОЖЕНИЯ

Annex 1



INITIAL ANALYTIC GEOMETRY FORMULAS



$$A(BRG, RNG) = A(B, R)$$
$$\Longrightarrow \begin{cases} x_A = R \cdot \sin B \\ y_A = R \cdot \cos B \end{cases}$$

$$(L) \equiv (RML) \equiv (A_1 A_2)$$
$$\Longrightarrow (L): y_L \cdot (x - x_1) - x_L \cdot (y - y_1) = 0$$

$$\boldsymbol{CPA = OD = \left|\overrightarrow{OD}\right| = \sqrt{x_D{}^2 + y_D{}^2}}$$

$$\boldsymbol{TCPA = \frac{A_2 D}{A_1 A_2} \cdot \Delta t}$$

$$\boldsymbol{SPD_A = 10 \cdot \sqrt{x_2'^2 + y_2'^2}}$$

$$\boldsymbol{CSE_A = \left(\widehat{Oy, \overrightarrow{OA_2'}}\right) = arctan \frac{x_2'}{y_2'}}$$

Annex 3

FINAL ANALYTIC GEOMETRY FORMULAS

$CPA = OD = \lvert\overrightarrow{OD}\rvert = \sqrt{x_D{}^2 + y_D{}^2}$	$\boldsymbol{CPA = \dfrac{\lvert R_1 \cdot R_2 \cdot \sin(B_1 - B_2)\rvert}{\sqrt{R_1{}^2 + R_2{}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}}$
$TCPA = \dfrac{A_2D}{A_1A_2} \cdot \Delta t$	$\boldsymbol{TCPA = \dfrac{\sqrt{R_2{}^2 - \dfrac{[R_1 \cdot R_2 \cdot \sin(B_1 - B_2)]^2}{R_1{}^2 + R_2{}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}}{\sqrt{R_1{}^2 + R_2{}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}} \cdot \Delta t}$
$SPD_A = 10 \cdot \sqrt{x_2{}'^2 + y_2{}'^2}$	$\boldsymbol{SPD_A = 10 \cdot \sqrt{\left(R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \dfrac{V}{10} \cdot \sin H\right)^2 + \left(R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \dfrac{V}{10} \cdot \cos H\right)^2}}$
$CSE_A = \left(\widehat{Oy, \overrightarrow{OA_2'}}\right) = arctan \dfrac{x_2{}'}{y_2{}'}$	$\boldsymbol{CSE_A = arctan \dfrac{R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \dfrac{V}{10} \cdot \sin H}{R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \dfrac{V}{10} \cdot \cos H}}$

THE DEFINED CONDITIONS

Vuong Hai

Quadrant	***I***	***II***	***III***	***IV***
x	$x \geq 0$	$x \geq 0$	$x \leq 0$	$x \leq 0$
y	$y \geq 0$	$y < 0$	$y < 0$	$y \geq 0$
$\sin BRG$	$+$	$+$	$-$	$-$
$\cos BRG$	$+$	$-$	$-$	$+$
$\tan BRG$	$+$	$-$	$+$	$-$
$\boldsymbol{BRG}$	$\arcsin z$ $\arccos z$ $\mathbf{arctan}\ \boldsymbol{z}$	$\arcsin z + 90°$ $\arccos z$ $\mathbf{arctan}\ \boldsymbol{z} + \mathbf{180°}$	$180° - \arcsin z$ $360° - \arccos z$ $\mathbf{arctan}\ \boldsymbol{z} + \mathbf{180°}$	$\arcsin z + 360°$ $360° - \arccos z$ $\mathbf{arctan}\ \boldsymbol{z} + \mathbf{360°}$

IMPLEMENTATION OF A COMPUTING APPLICATION

Vuong Hai

INPUT DATA

The own ship

HDG	SPD
240.6	12.4
(degrees)	*(knots)*

The target A

		BRG	RNG
Moment t_1	0	240.4	9.63
Moment t_2	6	237.4	9.58
	(minutes)	*(degrees)*	*(nm)*

OUTPUT DATA

The target A

From the initial formulas

CPA	TCPA	CSE	SPD
9.5545608	8.2838204	217.50316	12.775235
(nm)	*(minutes)*	*(degrees)*	*(knots)*

From the final formulas

CPA	TCPA	CSE	SPD
9.5545608	8.2838204	217.50316	12.775235
(nm)	*(minutes)*	*(degrees)*	*(knots)*

Comparison data	ARPA data	Analytic geometry
CPA	$9.56\ (nm)$	$9.555\ (nm)$
$TCPA$	$8.1\ (min)$	$8.284\ (min)$
CSE	$217.4°$	$217.503°$
SPD	$12.9\ (kn)$	$12.775\ (kn)$